Calibrating the Average Growth Factor Model
Define the growth factor for zone i :

$$F_i^k = \frac{O_i^*}{O_i^k}$$

and the average growth rate between zones i and j :

$$F_{ij}^k = \frac{F_i^k + F_j^k}{2}.$$

Then the iteration for this model is:

$$T_{ij}^{k+1} = T_{ij}^k \times F_{ij}^k$$

Example Data

$$T_{ij}^0 = \begin{array}{cccc|c} 0 & 12 & 10 & 18 & 40 \\ 12 & 0 & 14 & 6 & 32 \\ 10 & 14 & 0 & 14 & 38 \\ 18 & 6 & 14 & 0 & 38 \\ \hline 40 & 32 & 38 & 38 & 148 \end{array}$$

Assume that the predicted originations are:

$$O_i^* = [80, 48, 114, 38]$$

Take the convergence criterion to be 5% ($\alpha_L = 0.95$, $\alpha_H = 1.05$).

Iteration 1
Zonal growth rates

$$\begin{aligned} F_i^o &= O_i^*/O_i^0 \\ &= [80, 48, 114, 38] \div [40, 32, 38, 38] \\ &= [2, 1.5, 3, 1] \end{aligned}$$

Average growth rates:

$$F_{ij}^0 = \begin{bmatrix} 2 & 1.75 & 2.5 & 1.5 \\ 1.75 & 1.5 & 2.25 & 1.25 \\ 2.5 & 2.25 & 3 & 2 \\ 1.5 & 1.25 & 2 & 1 \end{bmatrix}$$

Example: the $(1,2)$ element of F_{ij}^0 is

$$\begin{aligned} F_{12}^0 &= \left(F_1^0 + F_2^0\right)/2 \\ &= (2 + 1.5)/2 \\ &= 3.5/2 \\ &= 1.75 \end{aligned}$$

and the new trip matrix is

$$\begin{aligned} T_{ij}^1 &= T_{ij}^0 \times F_{ij}^0 \\ &= \begin{array}{cccc|c} 0 & 21 & 25 & 27 & 73 \\ 21 & 0 & 31.5 & 7.5 & 60 \\ 25 & 31.5 & 0 & 28 & 84.5 \\ 27 & 7.5 & 28 & 0 & 62.5 \\ \hline 73 & 60 & 84.5 & 62.5 & 280 \end{array} \end{aligned}$$

(using term-by-term multiplication).
Convergence test:

Target	80	48	114	38
Actual	73	60	84.5	62.5
Error ratio	1.09589	0.8	1.34911	0.608

Obviously, no convergence

Iteration 2
The new zonal growth factors are the error ratios from the previous test:

$$F_i^1 = [1.09589, 0.8, 1.34911, 0.608]$$

New average growth factor matrix:

$$F_{ij}^1 = \begin{bmatrix} 1.09589 & 0.947945 & 1.2225 & 0.851945 \\ 0.947945 & 0.8 & 1.07456 & 0.704 \\ 1.2225 & 1.07456 & 1.34911 & 0.978556 \\ 0.851945 & 0.704 & 0.978556 & 0.608 \end{bmatrix}$$

New T_{ij} matrix:

$$T_{ij}^2 = T_{ij}^1 \times F_{ij}^1$$

$$= \begin{array}{cccc|c} 0 & 19.9068 & 30.5625 & 23.0025 & 73.4719 \\ 19.9068 & 0 & 33.8485 & 5.28 & 59.0354 \\ 30.5625 & 33.8485 & 0 & 27.3996 & 91.8106 \\ 23.0025 & 5.28 & 27.3996 & 0 & 55.6821 \\ \hline 73.4719 & 59.0354 & 91.8106 & 55.6821 & 280 \end{array}$$

Convergence check:

Target	80	48	114	38
Actual	73.4719	59.0354	91.8106	55.6821
Error ratio	1.08885	0.813072	1.24169	0.682446

No convergence.

Iteration 3

$$F_i^2 = [\ 1.08885 \quad 0.813072 \quad 1.24169 \quad 0.682446\ ]$$

$$F_{ij}^2 = \begin{bmatrix} 1.08885 & 0.950962 & 1.16527 & 0.885649 \\ 0.950962 & 0.813072 & 1.02738 & 0.747759 \\ 1.16527 & 1.02738 & 1.24169 & 0.962066 \\ 0.885649 & 0.747759 & 0.962066 & 0.682446 \end{bmatrix}$$

$$T_{ij}^3 = \begin{array}{cccc|c} 0 & 18.9307 & 35.6136 & 20.3721 & 74.9164 \\ 18.9307 & 0 & 34.7753 & 3.94817 & 57.6541 \\ 35.6136 & 34.7753 & 0 & 26.3602 & 96.749 \\ 20.3721 & 3.94817 & 26.3602 & 0 & 50.6805 \\ \hline 74.9164 & 57.6541 & 96.749 & 50.6805 & 280 \end{array}$$

Convergence check:

Target	80	48	114	38
Actual	74.9164	57.6541	96.749	50.6805
Error ratio	1.06786	0.832552	1.17831	0.749795

Still no convergence

Iteration 4

$$F_i^3 = [\ 1.06786 \quad 0.832552 \quad 1.17831 \quad 0.749795\ ]$$

$$F_{ij}^3 = \begin{bmatrix} 1.06786 & 0.950204 & 1.12308 & 0.908826 \\ 0.950204 & 0.832552 & 1.00543 & 0.791173 \\ 1.12308 & 1.00543 & 1.17831 & 0.964051 \\ 0.908826 & 0.791173 & 0.964051 & 0.749795 \end{bmatrix}$$

$$T_{ij}^4 = \begin{array}{cccc|c} 0 & 17.988 & 39.997 & 18.5147 & 76.4997 \\ 17.988 & 0 & 34.9641 & 3.12368 & 56.0757 \\ 39.997 & 34.9641 & 0 & 25.4126 & 100.374 \\ 18.5147 & 3.12368 & 25.4126 & 0 & 47.051 \\ \hline 76.4997 & 56.0757 & 100.374 & 47.051 & 280 \end{array}$$

Convergence check:

Target	80	48	114	38
Actual	76.4997	56.0757	100.374	47.051
Error ratio	1.04576	0.855985	1.13576	0.807634

Still no convergence

You can see that we are converging, but rather slowly. It turns out that convergence is attained at iteration 9, when we have

$$F_i^8 = [\ 0.996703 \quad 0.94 \quad 1.05272 \quad 0.941034\ ]$$

$$F_{ij}^8 = \begin{bmatrix} 0.996703 & 0.968352 & 1.02471 & 0.968869 \\ 0.968352 & 0.94 & 0.996362 & 0.940517 \\ 1.02471 & 0.996362 & 1.05272 & 0.996879 \\ 0.968869 & 0.940517 & 0.996879 & 0.941034 \end{bmatrix}$$

$$T_{ij}^9 = \begin{array}{cccc|c} 0 & 14.5795 & 51.6665 & 14.3279 & 80.5738 \\ 14.5795 & 0 & 34.0183 & 1.75437 & 50.3522 \\ 51.6665 & 34.0183 & 0 & 23.6535 & 109.338 \\ 14.3279 & 1.75437 & 23.6535 & 0 & 39.7357 \\ \hline 80.5738 & 50.3522 & 109.338 & 39.7357 & 280 \end{array}$$

Convergence check:

Target	80	48	114	38
Actual	80.5738	50.3522	109.338	39.7357
Error ratio	0.992878	0.953286	1.04264	0.956318

All of which are between 0.95 and 1.05 for 5% convergence. So T_{ij}^9 is our estimate of the trip distribution matrix.