Consider a profit-maximizing competitive firm operating in the short run with

\[\begin{align*}
AVC(x) &= 80 - 2x + 0.1x^2 \\
SRMC(x) &= 80 - 4x + 0.3x^2 \\
TFC &= 200
\end{align*}\]

where: \(x\) = output; \(AVC\) = average variable costs; \(SRMC\) = short-run marginal costs; \(TFC\) = total fixed costs. If the firm faces a price of 80 for its product:

(a) Show that its optimal output is 13.333.

(b) Calculate its profits.

(c) Would you advise it to shut down and cease production?

**Answer Sketch:** For this firm:

(a) Since you’re given an answer, you just need to verify that it’s correct, which in this case means checking that \(SRMC = p\). Here we have

\[SRMC(13.33) = 80 - (4 \times 13.333) + (0.3 \times 13.333^2) = 79.999\]

close enough to the price to allow us to conclude that the firm is behaving like a profit-maximizing competitor.

(b) Profits are Total Revenue minus Total Cost. Total Revenue is

\[TR = p \times x = 80 \times 13.333 = 1066.64\]

Total Cost is Total Variable Cost + Total Fixed Cost. Total Variable Cost is Average Variable Cost \(\times\) quantity. Then we calculate

\[AVC(13.333) = 80 - (2 \times 13.333) + (0.1 \times 13.333^2) = 71.111\]

and total variable costs are 71.111 \(\times\) 13.333 = 948.12296. Finally we add in fixed costs (200) to find that total costs are 948.12296 + 200 = 1148.1230; hence profits are 1066.64 - 1148.1230 = -81.483.

(c) Profits are negative: does this mean the firm should shut down? Not necessarily. Note that revenues are more than total variable costs: this means that the firm is more than covering its variable costs, and making some contribution to fixed costs. Alternatively: if it stays in business, the firm incurs a loss of 81.5. If it were to shut down its loss would be 200 (i.e., total fixed costs). Clearly the lower loss is preferable, so it should not shut down.
2. Define the Equitable Allocation as a distribution of goods and services in which each individual receives an equal (physical) share. Assume that all goods and services are perceived as beneficial (i.e., have positive utility).

   (a) If a society is composed of clones (identical individuals) is the Equitable Allocation Pareto-Optimal? (Explain briefly).
   (b) What if the members of the society are not clones?

   **Answer Sketch:**

   (a) In a society of clones the Equitable Allocation is clearly Pareto-Optimal, since any attempt to take from one individual and give to another must necessarily make the first individual worse off. Thus there is no re-allocation capable of making one individual better off while leaving everyone else indifferent, and the Equitable Allocation is Pareto-Optimal.

   (b) However, if the people are not clones then it all depends on preferences. If I hate x and love y, and your preferences are the opposite, then clearly there’s a trade that can make us both better off: I give you some of my x and you give me some of your y. (The exact terms are negotiable). This is an improvement for both of us, showing that the equitable allocation can be improved on. Since we normally expect people to have varying preferences, this strongly suggests that the Equitable Allocation is not Pareto-Optimal. (This is probably something you should bear in mind when someone suggests equal distribution as a criterion of equity).

3. Two spatially separated firms produce the same product with the same 2-input non-L-shaped production function (isoquants). Both firms are input-price takers. One input is available everywhere at the same price. The other input must be transported to the firms’ locations. Both firms face the same transport costs, but the first firm is located closer to this input.

   What can you say about the two firms’ relative utilizations of the two inputs?

   **Answer Sketch:** What this comes down to is that while both firms face the same prices for the first input, the second firm pays more for the second input. Therefore its isocost line is steeper than that of the first firm and will use proportionately less of input 2.

4. Two firms are located at either end of a 1-mile long road. The firm on the left posts a factory (free-on-board) price of 4 per ton, while the one on the right has a factory price of 3.9 per ton. Transport costs are asymmetric: for the firm on the left they are 0.1 per ton-mile, while for the firm on the right transport costs are 0.2 per ton-mile.

   (a) Write down a formula for the delivered price for the firm on the left.
   (b) Write down a formula for the delivered price for the firm on the right.
   (c) What can you say about the market areas of the two firms?

   **Answer Sketch:** This is an instance where notation matters. If you use the same symbol (say $s$) for distance, then it has to mean the same thing every time you use it. Put another way, in any spatial setting, you need to establish a coordinate system and stick to it. For this problem, a coordinate system is just points on a line, and we may as well measure distances from the location of the firm on the left, which we call point $s = 0$.

   (a) The delivered price of the firm on the left is FOB price + transport cost to any location. Since we are measuring distances from its location, this is: $DP_l(s) = 4 + 0.1s$ where $s$ is distance from the firm on the left.
(b) For the firm on the right it’s basically the same thing: delivered price is FOB price + transport costs. But if we want to ship to any point \(s\) between the two firms then, given our coordinate system, the distance shipped from the firm on the right is \(1 - s\) (draw a picture). So \(DP_2(s) = 3.9 + 0.2(1 - s) = 4.1 - 0.2s\).

(c) To find the market areas, we find the point in space where the two firms are competitive in delivered price, by solving the two delivered price equations. We have

\[
\begin{align*}
4 + 0.1s &= 4.1 - 0.2s \\
0.3s &= 0.1 \\
s &= 1/3
\end{align*}
\]

which says that the firm on the left sells from 0 to \(1/3\), while the firm on the right sells from \(1/3\) to 1.0. Note that we don’t know that there will actually be any sales in the market areas; that depends on demands. But no matter what the demands, we’d not expect the firm on the right to sell in the areas \([0,1/3]\) (or the firm on the left to sell in \([1/3,1]\)).

5. An activity \((x)\) has private marginal costs of

\[MC_p(x) = 3 + 1.5x\]

and gives rise to private marginal benefits of

\[MB_p(x) = 15 - x\]

and social marginal benefits of

\[MB_s(x) = -0.5x\]

(a) Show that the private optimum is \(x = 4.8\)

(b) Show that the social optimum is \(x = 4\). (Hint: do you need to actually sum the two benefit curves?)

**Answer Sketch:** This of course is a case of negative externalities (\(MB_s\) is always negative), so we expect that the social optimum will be less than the private one.

(a) For the private optimum we equate private costs and benefits:

\[
\begin{align*}
3 + 1.5x &= 15 - x \\
2.5x &= 12 \\
x &= 12/2.5 = 4.8
\end{align*}
\]

as claimed. Alternatively, since you’re given the supposed answer, you can just verify that it’s correct: \(MC_p\) must equal \(MB_p\). In this case \(MC_p(4.8) = 3 + (1.5 \times 4.8) = 10.2\), while \(MB_p = 15 - 4.8 = 10.2\), showing that 4.8 is indeed the private optimum.

(b) For the social optimum, we don’t actually need to do the vertical addition: all we need to do is verify that at the give output, total marginal benefits equal marginal costs. We have

\[
\begin{align*}
MC_p(4) &= 3 + (1.5 \times 4) = 9.0 \\
MB_p(4) &= 15 - 4 = 11 \\
MB_s(4) &= -0.4 \times 4 = -2
\end{align*}
\]
So that total marginal benefits are $11 - 2 = 9$, which is equal to private marginal cost. Alternatively, if you did want to do the addition, $MB_T(x) = MB_p(x) + MB_s(x) = 15 - x - 0.5x = 15 - 1.5x$. Then equate $MB_T$ and marginal cost:

\[
15 - 1.5x = 3 + 1.5x \\
12 = 3x \\
x = 4
\]

as before.

6. Assume that the only function of street lights is to deter crime against the property of householders. Since this is a purely private benefit, why couldn’t we rely on the householders to provide street lights themselves, instead of having them provided by municipalities?

**Answer Sketch:** You can think of this in a number of ways: the street lights create a (positive) externality, since if I put up a light near my house, I thereby create increased safety for you (joint product), the next-door neighbor, a benefit that I don’t realize (market failure). Alternatively, the benefits of street lights are a non-excludable public good (in the technical sense). As such, they create a free-rider problem: if I’m asked to contribute to the cost of the lights according to the (expected) benefits I receive, my best strategy is to say that my value is zero, and hope that someone else will be honest and cough up the cost of installation. (As far as I can see, it’s not a question of maintenance of the lights, or the fact that it may be dangerous to replace a burned-out bulb: if that was thought to be an issue, I could always contract with a private light-maintenance company to repair/replace the lights when they burned out, just as I contract with someone to mow my lawn.)

7. Suppose that a city provides garbage collection services under the following cost conditions:

\[
\begin{aligned}
\text{Total Cost} & : \quad TC(g) = 4g + 2g^2 \\
\text{Average Cost} & : \quad AC(g) = 4 + 2g \\
\text{Marginal Cost} & : \quad MC(g) = 4 + 4g
\end{aligned}
\]

(where $g$ is the volume of garbage collected). Based solely on cost considerations, could this service be privatized and provided by many small firms (assume that private providers would face the same cost curves)?

**Answer Sketch:** Look at the average and marginal cost curves. They both have the same intercept (4), but the slope of the MC curve is higher. In other words, in this industry, $MC > AC$, indicating DRTS. Therefore a private firm could behave like a competitor here, with its price ($= MC$) more than covering it’s average cost. (So total revenue, which will be $MC \times \text{quantity}$, will be more than total cost, $AC \times \text{quantity}$). If this is so, then, at least on cost considerations, there’s no reason that the service couldn’t be privatized. (Are there other considerations? Or does this show, even if we expand the problem to be broader than just cost, there’s still no reason for the city to provide garbage services? Of course, it might turn out that my cost curves were simply wrong, and that the service is subject to IRTS).

8. A profit-maximizing competitive firm produces 2 units of output under constant returns to scale at an average cost of 4, where this includes all economic costs except transportation and land-rent costs. It sells all output at price $p = 15$ at a single city market: transportation costs are $1.5s$ per unit of output, where $s$ is the distance to the market. To produce its output the firm requires 1 unit of land, whose rent at distance $s$ from the market is $R(s)$.
(a) In free-entry industry competitive equilibrium, what is the firm’s bid-rent function for land?

(b) Suppose that residents in the city have bid-rents

\[ R_r(s) = 20 - 0.8s \]

If these are the only two groups competing for land, what will be the equilibrium pattern of land-use in the city?

\[ A N S W E R \ S K E T C H: \] The firm’s total revenue from selling its 2 units of output is \( 15 \times 2 = 30 \). Total costs are \( 4 \times 2 = 8 \) for the non-transport, non-land costs, \( 1.5 \times 2 \times s = 3s \) per unit distance in transport costs, and \( 1 \times R(s) \) for land costs.

(a) In free-entry equilibrium (economic) profits are driven to zero, hence must satisfy

\[
0 = 30 - 8 - 3s - R(s) \\
= 22 - 3s - R(s)
\]

Hence

\[ R(s) = 22 - 3s. \]

(b) Land goes to the highest bidder. At the city center \( s = 0 \) bid rents are 22 by the producer and 20 for residents; hence firms locate close to the center. The switch-over to residential use comes where the two bids rents are the same, so we solve

\[ 22 - 3s = 20 - 0.8s \]

So

\[ 2 = -0.8s + 3s = 2.2s \]

Hence \( \hat{s} = 2/2.2 = 0.909 \) and residences begin about 0.91 miles from the CBD. To find the extent of the city (assuming that there is no other, eg agricultural) use for land, we find when city residents will bid zero for land, ie we solve

\[ 0 = 20 - 0.8s \]

Hence \( s_f = 25 \) miles. (It’s a very extensive city).