Some Facts Needing Explanation

- As we get further away from an activity center, land rents decline.
- As we get further away from an activity center, lot sizes increase.
- As we get further away from an activity center, population density falls.
- In the US at least, we observe a good deal of spatial segregation by class: the rich tend to live in the suburbs while the poor tend to live in the central cities.

Can we develop a way of understanding these facts? And what about city edges (boundaries)? Can we understand why they are where they are?

Spatial Setting (I)

- All activity takes place on a flat featureless von Thunen plain. (This means that geography — rivers, mountains etc — is irrelevant).
- There is a small city in the center of the plain. The city is made up of:
  - A single central business district (CBD) which we will take to be a point. This defines the monocentric city.
  - Suburbs surrounding the CBD.
- Around the city is a “hinterland” devoted to agriculture.
- We measure all distances $s$ as distances from the CBD.

Spatial Setting (II)

- The von Thunen assumption also implies that direction is irrelevant.
- So we also know that the city will be circular.
- The distance from the CBD to the edge of the city is denoted $s_f$.
- This marks the boundary (“fringe”) between suburban and agricultural land-uses.
Model: Spatial Setting (III)

Note that our interest here is the mirror-image of the von Thunen model:

- Here, we are interested in land use within the city; in the von Thunen model we were interested in land use outside the city.
- Here, land-use outside the city is a single agricultural “hinterland”; in the von Thunen model, land inside the city was implicitly considered as a single “city” land-use.
- In effect, we now propose to open up and study land use in the city.

Economy

Urban sectors:
- Land sector: we denote lot sizes in the city at distance $s$ from the CBD by $q(s)$.
- City transport sector: a single mode of transport is available in any direction.
- Goods sector: besides land, there is a single composite good $z$.

Prices
- City land price (called the land rent) at distance $s$ from the CBD is $R(s)$.
- Agricultural land rent is $R_A$ everywhere in the hinterland.
- Transport: available at a cost of $k$ per round-trip mile.
- The composite good is available everywhere (it is ubiquitous) at a price of $p_z$.

Individuals

- In contrast to the firms that we studied in our von Thunen model (where our assumptions eliminated most decisions, eg on input requirements), the fundamental feature about individuals is that they make choices.
- Specifically, they decide on the allocation of their resources between land and the composite good.
- So we need to study how this will take place.
- Note: the following discussion is an introduction to the standard economic theory of individual choice, geared to our interest in land. You can find more on this in any microeconomics text, where the discussion is generally phrased in terms of arbitrary unnamed goods. Otherwise, the discussions are exactly the same.

Preferences (I)

- Since people are deciding on the allocation of their resources between land ($q$) and everything else (the composite good, $z$), we focus on different land-and-$z$ combinations, called bundles of commodities, where a bundle consists of a quantity of land ($q$) and a quantity of $z$.
- Individuals’ allocation decisions will depend on two things: the individual’s preferences/feelings/psychology regarding land versus other commodities (the composition of the bundles), and the resources that the individual has available to satisfy those preferences.
- We begin by looking at preferences.
In our monocentric city, individuals must decide on their allocations between land and the composite commodity. This is a 2-good setting.

- If we set up a pair of axes with \( z \) on the vertical axis and \( q \) on the horizontal axis, then each bundle can be represented by a point in this plane.
- In the figure, bundle A has \( q_1 \) units of land and \( z_1 \) units of the composite commodity.
- Bundle B has less land, but more \( z \).

Preferences (III)

- Individuals choose between different possible bundles of goods.
- Everything we know about individuals suggests that they regard bundles as substitutable: two bundles with different quantities of the two goods can appear to an individual as equally desirable.
- Different individuals will have different preferences: two bundles that one individual regards as equally desirable might not be equally desirable for someone else.
- In what follows, we will discuss the preferences of an arbitrary individual. But bear in mind that individuals can be different. This can affect the shapes of our diagrams, but not their fundamental logic (see slide 14).

Indifference Curves (I)

- Let’s start with some base bundle A.
- We now identify all those bundles that this individual regards as equally desirable (to the base bundle A). Suppose that two of them are bundles B and C.
- We connect up the dots representing all these equally-desirable bundles.
- We refer to this representation as an *indifference curve*, denoted \( I \) in the figure.

Indifference Curves (II)

- By definition, any two bundles on the same indifference curve are regarded by this individual as equally desirable.
- By the same token, there is some indifference curve going through any particular bundle. That is, we can find bundles that the individual regards as equally desirable to *that* bundle.
- In advanced courses you will learn that indifference curves are a graphical way of visualizing a more formal (mathematical) way of representing an individual’s preferences, called a utility function. It is a fundamental fact that most preference patterns can be represented by some utility function, and hence (in the 2-good case) by some indifference curve.
We can now see how our diagrams can represent different preferences.

- In the figure opposite, the two panels show an indifference curve for two different individuals (X and Y). Note that they are drawn differently.
- Let’s ask: suppose we take away $\Delta q$ of land from both of them, and then offer enough $z$ to make up for it (i.e., keep them on the same indifference curve).
- For individual X, we would need to offer $\Delta z_1$ more of the composite good. By contrast, for individual Y, we would need to offer only $\Delta z_2$ of $z$.
- This says that X regards land as very important for his well being: if he has to give up some land, it takes a lot of $z$ to make up for it.
- By contrast, Y is less fixated on land: if we take away some land, we will need to give her more $z$ to make up for it, but not nearly as much as we needed to give X.

Suppose our individual prefers more land to less land, and more $z$ to less $z$.

- Now consider bundles A and B.
- Bundle B has more land than bundle A, and more $z$ too.
- So B must be preferred to A, and cannot be on the same indifference curve as A.
- But there will be some indifference curve through B.
- So being on a higher indifference curve is better than being on a lower one.

The upshot is that an individual’s preferences (as regards land and $z$) can be represented by a family of indifference curves.

- Any single indifference curve represents bundles that the individual regards as equally good.
- Being on a higher indifference curve is better than being on a lower one, as indicated by the little arrow (direction of increasing satisfaction or well-being).
Indifference Curves and Preferences (III)

It can be shown that indifference curves have the following properties:

1. They slope downwards, as long as the two goods making up bundles are both regarded as good (more is better).
2. They cannot cross.
3. They are dense in commodity space. This just means that there is an indifference curve through every bundle.
4. Higher indifference curves (curves to the north-east) are preferred to lower ones.

Note that we do not assume that the indifference curves all have the same general shape, even if we’re drawing them that way for convenience.

Indifference Curves and Choice

Under our assumptions it makes sense to say:

- When selecting a consumption bundle, the individual will try to select what he or she perceives as the best (affordable) bundle.
- Equivalently, she will try to get on the highest (affordable) indifference curve.
- Equivalently, given that indifference curves are visualizations of utility functions, we say that an individual is a utility maximizer (subject to affordability).

Affordability (I)

- What prevents an individual from reaching an arbitrarily high indifference curve?
- Obviously, the fact that goods (here, land and the composite commodity) must be paid for; and the individual does not have infinite resources for this.
- So let’s study which bundles (combinations of q and z) are affordable for our individual.

Affordability (II)

- Consider an individual located s miles from the CBD.
- We assume that land there has a price (rent) of $R(s)$, per acre, say.
- As we’ve noted, z is ubiquitous with price $p_z$, independent of location.
- We assume that the individual behaves as if prices will not change, no matter what she decides.
- We say that she is a price-taker.
The individual located at distance $s$ faces prices $R(s)$ and $p_z$ for land and the composite commodity, respectively.

We assume that she has (wage) income $M$ to divide up between the two goods.

Therefore, any affordable bundle must satisfy

$$R(s)q + p_z z \leq M$$

— total expenditure on land ($R(s)q$) plus total expenditure on everything else ($p_z z$) must be less than or equal to available income ($M$).

It turns out that in our two-good setting, an individual will never spend less than her available income.

This is because if she did so, and she is a utility maximizer, she could do better (reach a higher indifference curve) by using the unspent income to purchase a bit more of each of the two goods.

Note that this depends on the composite good $z$ really representing “everything else”, so it would include such things as saving for retirement etc.

So we now know that a best affordable bundle will exhaust her resources, i.e., will satisfy

$$M = R(s)q + p_z z$$

This is her Budget Constraint.

Can we visualize an individual’s budget constraint? That is, can we plot the individual’s affordable combinations of $q$ and $z$, given prices ($R(s)$ and $p_z$) and income ($M$)?

Let’s examine the quantity of $z$ that an individual can afford, if she consumes $q$ units of land.

To find this, we’ll solve the budget constraint for $z$ as a function of $q$.

The result is:

$$z = \frac{M}{p_z} - \frac{R(s)}{p_z} q$$

This is the equation of a straight line.

Its slope is $-R(s)/p_z$, called (minus) the price ratio, and which is negative. So the budget constraint is a downward-sloping straight line.

If our individual buys no land, then the maximum $z$ she can afford is $M/p_z$.

If she purchases no $z$, then the maximum acreage of land she can afford is $M/R(s)$.

So her budget constraint is a straight line connecting $M/p_z$ on the $z$ axis with $M/R(s)$ on the $q$ axis.

You can also see this from the equation on the previous slide.
Income changes

- Suppose income increases to \( M' \) while nothing else changes.
- The new budget constraint is \( M' = R(s)q + p_z z \). This is still a straight line, with the same slope \(-R(s)/p_z\) as before, since we are assuming that nothing else changes.
- But now our individual can afford more land and more of the composite good.
- So the new budget constraint is parallel (same slope) to the old one, but higher up.

Price changes (I)

- Suppose the price of \( z \) falls to \( p_z' \), while nothing else changes (income is again \( M \)).
- If she buys only land, nothing changes: she can still afford \( M/R(s) \) acres.
- But if she buys only \( z \), she can now afford more: \( M/p_z' \)
- So when \( p_z \) falls, her budget constraint pivots outwards around \( M/R(s) \).
- If \( p_z \) rises, the pivot will be inwards, but still a pivot around \( M/R(s) \).

Price changes (II)

- If the land rent at \( s \) increases to \( R'(s) \), then the budget constraint would pivot inwards around \( M/p_z \) to \( M/R'(s) \), as in the figure.
- If the land rent at \( s \) decreases, then the pivot would still be around \( M/p_z \), but now outwards (not shown).

Individual Equilibrium

- Begin with the budget constraint.
- Now superimpose the indifference curves.
- The highest indifference curve consistent with the budget constraint will be tangent to the budget constraint.
- In the figure, her best bundle (at \( s \)) is \((q^*(s), z^*(s))\)
Equilibrium After Price Change

- Suppose the price of land at $s$ rises to $R'(s)$ (heavy budget line).
- The individual will choose a new best bundle.
- As shown, she will decide to consume less land and more $z$ ($q^{**}(s)$ and $z^{**}(s)$, respectively).
- This is typical behavior: when a price increases, the response is usually to consume less.

Suppose income rises to $M'$ (heavy budget line).
- As shown in the figure, the individual increases her consumption of both land and $z$ to $q^{***}(s)$ and $z^{***}(s)$, respectively.

Superior and Inferior Goods

- In the previous slide, an increase in income led to more land (and more $z$) being consumed. But this is not the only possibility.
- It could happen that in with increased income, our individual decides to consume less of one of the goods.
- When the response to increased income is to increase consumption of a good, we say that the good is a superior (sometimes normal) good.
- When the response to increased income is to reduce consumption of a good, we say that they good is an inferior good.
- Note that this is intended to carry no moral connotation: it’s simply meant to describe behavior. (But the terminology is perhaps a bit unfortunate).

Conclusion

- This concludes our brief survey of individual choice.
- You can find more information in any microeconomics text.
- We now return to individual behavior in the context of our moncentric city, and try to answer the questions raised at the beginning of these notes.