Extensions of the Moncentric City Model

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Suppose our urban area is served by two modes of transport.

We will assume that both modes are equally available in all directions.

People choose the least-cost mode, where “cost” here is “generalized cost”, including both out-of-pocket costs and the time costs needed to make a trip.
Specifically, we assume:

- Mode 1: total cost: $C_1(s) = k_{11}s$
- Mode 2: total cost: $C_2(s) = k_{20} + k_{21}s$

The intuition here is that mode 2 has a fixed cost which must be incurred even if you don’t make any trips. We could think of this as the costs of owning a car (capital costs, insurance costs etc) whether you drive it or not. Mode 1 on the other hand has no fixed costs. We can perhaps think of it as transit.

- But once the mode-2 fixed costs have been incurred, we will assume mode 2 has lower per-mile costs: $k_{21} < k_{11}$. 
The low-cost picture is as in the figure: mode 1 will be used up to distance $s^*$ and mode 2 thereafter.

The crossover point is where the two costs are equal:

$$k_{11}s = k_{20} + k_{21}s$$

$$(k_{11} - k_{21})s = k_{20}$$

$$s^* = k_{20}/(k_{11} - k_{21})$$

This could explain why transit isn’t used much for longer suburban commutes.
Multimodal Transport (IV)
To see the impact on land rents, suppose mode 2 is the new mode. This induces a new disposable income line $Y_2(s) = (M - k_{20}) - k_{21}s$. At any distance $s$ people will choose the mode that gives them the higher disposable income. Then:

- At the CBD ($s = 0$) land rent is unchanged.
- Between $s = 0$ and the modal crossover point $s^*$ land rent is unchanged.
- At distances further away from the CBD than $s^*$, people will switch to the new mode.
- For one of the switchers at $s_1$ the budget constraint changes from $B$ to $A$. The economic effect is that land rents at $s_1$ increase.
- The city expands.
If an all-mode-2 city introduces a no-fixed-cost mode like mode 1 (starts a transit system):

- At the CBD, and up to the modal cross-over point, land rents increase.
- Beyond the crossover point, land rents are unchanged.
- The city fringe distance is unchanged.
Suppose our urban area has two CBDs (activity centers), which we can consider as located at either end of a line.

People must commute to one or another of the CBDs to earn income.

We can assume that each CBD and surrounding suburb supports a different single form (mode) of transportation (ie has different transportation costs).
Our previous arguments establish that land rents will decline as we go away from either CBD.

The figure opposite shows one possible result: $R^*_L(s)$ is the land rent if you commute to the left-hand CBD, and $R^*_R(s)$ is the land rent if you commute to the right-hand CBD.

Here, urbanites outbid farmers for all the land between the two centers. The observed rent pattern is the upper envelope of the individual rent curves.
Here is another possibility. In this case the farmers outbid urbanites for land between $s_1$ and $s_2$.

- The observed land-rent pattern has a flat portion.
- We get what might be thought of as green-space between the two centers.
Finding the Centers

- We know that rents decline as we go away from the nearest activity center, which we should probably take as an employment center, given our assumption that people need to commute to these centers to earn income.

- It is an interesting question, how to identify the various centers in an urban area, particularly large urban areas. A paper by Giuliano and Small (see references) discusses how they identified centers in the LA metro area.

- It might be interesting to try and apply their ideas to Columbus: one center is probably the area around the State House, but how far around it does the center extend? Another is probably the OSU campus, where the extent is presumably more straightforward. Are there others? What about in the suburbs?
The 2-Class Monocentric City

- Two classes of individuals:
  - Rich, with incomes \( M_R \).
  - Poor, with income \( M_P \).

- Obviously, we want \( M_R > M_P \).

- Within each class, identical individuals. But individuals’ tastes (utility functions) may differ as between the two classes.

- All use the same 1-mode transport system.

- Land is a *normal* good: as incomes go up (holding everything else constant) people want to consume more land.
We now have representatives from two classes potentially interested in the same plot of land.

Who gets the land?

Obvious principle: land goes to the person who bids higher for it.

So we need to see how much each class (ie a representative person of that class) will bid for land (at a given distance).

This is the same problem we faced in our discussion of the von Thunen model, but, as we’ll now see, it’s a bit more complicated.
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Bid Rent (I)
To analyze this, consider a person living $s_1$ miles from the CBD (see figure opposite). Her disposable income is $Y(s_1)$.

Question: What is the difference between budget constraints $A$ and $C$?

Answer: because it is steeper, $C$ represents a higher land price.

So to find the maximum bid for land for someone living at $s_1$ consistent with spatial equilibrium, we find the steepest line through $Y(s_1)$ that is just tangent to the spatial equilibrium indifference curve $I^*$. 
Bid Rent (III)

\[ Y(s) \]

\[ Y(s_1) \]

\[ z, s \]

\[ \text{A} \]

\[ \text{B} \]

\[ s_1 \]

\[ q \]
In the figure, this steepest budget line for someone living at $s_1$ consistent with spatial equilibrium is line $B$.

So the slope of $B$ represents the highest bid for land by someone at $s_1$, consistent with spatial equilibrium.

But of course, this is what we have been working with all along (the tangency between the indifference curve and the budget line through $Y(s_1)$).

The maximum bid for land (at some distance) that is consistent with spatial equilibrium is called the *Bid Rent* of land (at that distance).
We can now apply this to our 2-class city.

If both classes live in the city, we have 2 downward-sloping bid-rent curves.

There must be one point in space \((s^*)\) where the two groups live side-by-side. Call this the integrated district.

The question is: which bid-rent curve is that of the poor and which belongs to the rich?
2-Class City: The Integrated District (I)

Diagram showing relationships between variables with labels and arrows indicating points of interest.
We can use the integrated district at distance $s^*$ to identify the indifference curves for the two groups.

At $s^*$ the poor people must attain the poor-persons’ spatial equilibrium indifference curve $I_p$: this determines their budget constraint ($A$).

At $s^*$ the rich pay the same rent: their budget constraint is therefore parallel to $A$.

And since at $s^*$ the rich have more disposable income than the poor, and land is normal, they demand larger lot sizes than the poor.

This tells us that their budget constraint ($B$) must be tangent to the spatial-equilibrium indifference curve of the rich so that at that tangency, the rich consume more land at $s^*$.

The result is the rich people’s spatial equilibrium indifference curve $I_R$. 
2-Class City — Who Lives Where (I)
Now consider a point like $s_1$, closer to the CBD than the integrated district at $s^*$.

- A poor person living there must have a budget constraint tangent to $I_P$: this is line $A$.
- Suppose that the rich bid the same rent as the poor at $s_1$.
- Then their budget constraint would be line $B$ (parallel to $A$).
- But this cannot be an equilibrium, since it is not tangent to $I_R$.
- In order for the budget constraint to be tangent to $I_R$ it must pivot outwards, to $C$.
- But as compared to $B$ (and $A$), line $C$ represents a lower bid-rent (at $s_1$) since it is less steep.
- We therefore conclude that the poor outbid the rich at $s_1$. 
We conclude that in our 2-class open monocentric city:

- The poor live closer to the CBD.
- The rich live in the suburbs.

This seems to be consistent with the observed spatial pattern in the US. Exercise: show that if land is inferior (that is, as your income goes up you consume less land) then we get the reverse pattern: the rich live near the CBD and the poor live in the suburbs. This may be consistent with the spatial pattern in Europe around the time of the Industrial Revolution: the rich tended to live near the CBD — that was where the government, ie political power, was — and the poor lived in the suburbs, and trekked in every day to earn their livings.
The city will end when the bid-rents of the rich suburbanites \((R_R(s))\) fall below the bid-rents of the farmers \((R_A)\).

In the figure, this is \(s_f\).
The equilibrium pattern of land rents will therefore be the *outer envelope* of:

- $R_P(s)$ (the bid-rents of the poor);
- $R_R(s)$ (the bid-rents of the rich);
- $R_A$ (the bid rent of the farmers).

In the picture, this is the heavy line.