Industrial Land Use — the von Thunen Problem

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So far we have examined the land-use (location) decision of a single firm.

We now expand our focus to examine the land-use (location) decisions of many different firms, grouped into industries.
von Thunen’s Model

- J. von Thunen, in Der isolierte Stadt, 1826 (The isolated city) considered the locations of agricultural activities around an isolated city with no inter-regional trade.
- This reflected the setting for many early-19th-century German cities, before the development of an extensive railway network.
- All agricultural output needed to be transported (by horse and cart) to the city to be sold.
- von Thunen’s work contained not only a sophisticated theoretical model but also a careful empirical analysis of land productivity, output weight, carrying capacity (horsepower) of the available transport, and of the need to carry additional weight (fuel) to feed the horses.
Rough pattern of von Thunen’s results for 19-th century Germany:

1. Perishables
   (fruits/vegetables/milk)
2. Lumber (heavy: you want to minimize transport costs)
3. Other crops
4. Livestock, which are their own (weight-losing) transport

(von Thunen’s actual results were more detailed than this).
Let’s try to build on the spirit of von Thunen’s model to derive some insight into modern industrial land use.
We will assume that all activity takes place on a von Thunen plain: a flat featureless plain.

Geographical features are ignored.

We assume that there is a city located in the middle of this plain.

We will focus on land use outside the city, so we represent the city as a point in space.
The Transport Sector

- We will assume that transportation to and from the city is available everywhere.
- We will measure distances $s$ by the distance in any direction from the city (which, remember, we consider as a point in space).
- Transport costs will be on a ton-mile basis, and write $k$ for the cost per ton-mile.
- We will assume for simplicity that these costs apply to all industries on our plain.
- You should be able to see in principle how this can be generalized: all we would need to do is write $k_j$ for the transport cost per ton-mile applicable to a representative firm in industry $j$. 
Our story is about *competition for land* by the representatives of the various industrial sectors.

At any distance from the city, land could be occupied by any of the sectors.

Which sector will *actually* get land at that distance?

We will make a natural assumption here: *land goes to the highest bidder*, ie whoever is prepared to pay most for it.

If sector 1 is prepared to bid more for a plot of land than sector 2, then sector 1 will get that land.
So we need to ask: how much is a representative of each industry willing to pay (bid) for a plot of land located $s$ miles from the city?

To discuss this, we need to say something about why the firms do what they do, i.e., their behavioral motivation.

Our answer here is conventional: firms make their decisions in order to maximize profits.

Thus, the amount a firm would be willing to bid for land at some distance from the city will depend on the level of its profits.
We will distinguish two ways of measuring profits.

- **Accounting Profits**: The definition of accounting profit is exactly what you would expect: accounting profits are the difference between total firm revenues, and the costs incurred to realize those revenues:
  \[
  \text{Accounting Profits} = \text{Total Revenues} - \text{Total Costs}
  \]

- **Economic Profits**: Economic profits take accounting profits and subtract out the profits that *could be* achieved, if resources were deployed in their next-best possible way. So
  \[
  \text{Economic Profits} = \text{Accounting Profits} - \text{Next-Best Profits}
  \]
Before we see why this is a useful distinction, let’s look at an example.

Suppose there are two opportunities, called A and B, described as follows:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Revenues</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>Total Costs</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>Accounting Profit</td>
<td>18</td>
<td>25</td>
</tr>
</tbody>
</table>

Accounting profit is just the difference between total revenues and total costs, so $30 - 12 = 18$ for opportunity A, and $50 - 25 = 25$ for opportunity B.
Accounting vs Economic Profits

- To calculate economic profit, we note that if resources devoted to opportunity A had been devoted instead to opportunity B, they would have yielded an accounting profit of 25. And since there are only two opportunities, this is the next-best profit for A.
- To arrive at the economic profit for A we subtract this next-best profit (25) from A’s accounting profit: the result is $18 - 25 = -7$.
- Similarly for B: its next-best opportunity is the profits offered by A, so economic profit is $25 - 18 = 7$.
- The upshot is:

<table>
<thead>
<tr>
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<td>25</td>
</tr>
<tr>
<td>Economic Profit</td>
<td>$-7$</td>
<td>7</td>
</tr>
</tbody>
</table>
What does this tell us? The important thing here is the sign of the economic profits.

- Opportunity A has a negative economic profit. This tells us that if we’re out to maximize our returns, that we can do better (we should shift to opportunity B).
- On the other hand, opportunity B has a positive economic profit. This tells us that there is no better way available to make profits (the next-best opportunity yields less profits).
- On this data, we could predict that resources will be shifted from A to B.
Accounting vs Economic Profits (IV)

- We can also think of including the opportunity costs in the firm’s direct (out-of-pocket) costs.
- Then we would have:
  
  \[
  \text{Total Economic Costs} = \text{Total Direct Costs} + \text{Opportunity Cost}
  \]

- And then:
  
  \[
  \text{Economic Profits} = \text{Total Revenues} - \text{Total Economic Costs}
  \]
We now apply these ideas to our industrial land-use problem.

- We think of an industry as made up of many small identical firms.
- In the jargon, we will consider competitive industries.
- Therefore we can consider a representative firm in each industry (industrial sector).
- Each industry manufactures a single product, and sells that product only in the city (whose location is given).
We have said that the willingness of a firm to bid for land depends on its profit level.

But what will profit levels be?

The key to understanding this is that if a sector is profitable, entrepreneurs will enter that sector; if there are insufficient profits to be made, then entrepreneurs will leave that sector (and migrate to another).

This movement will change the industries’ profit levels, and hence their willingness to bid for land.
We will assume that nothing prevents a firm from entering a profitable sector, or leaving an unprofitable one.

In the jargon, there are no *barriers to entry* (or exit) for any industrial sector.

The most important barrier to entry found in practice is regulatory: some governmental agency restricts the ability of firms to enter (typically this question does not arise for leaving) an industrial sector.

Examples: in many cities, entry to the taxicab sector is regulated; before the Air Deregulation Act of 1976, entry into the passenger air transport market was regulated (by the Civil Aeronautics Board).

Our assumption is that there is Free Entry and Free Exit for all the industries that might locate on the plain.
Long-Run Profits

- If a sector has positive economic profits, entrepreneurs will be induced to enter the sector. This will reduce the profitability of the sector (more competition).
- If the sector has negative economic profits, entrepreneurs will leave the sector. This will enhance the profitability of the remaining firms (less competition).
- Our assumption of free entry and exit means that there are no reasons for these changes not to occur.
- The result is that entry and exit will eventually (i.e., in the long run) drive economic profits (but not necessarily accounting profits) to zero.
- This zero-profit outcome is often referred to as the long-run industry equilibrium for a competitive industry.
- We will focus only on the long run. We will not be discussing the path to equilibrium, i.e., how we actually get there.
Let’s return now to our discussion of economic activity in space.

- We will adopt a von-Thunen-like (agricultural) setting for our firms and industries.
- All inputs and output will be measured in weight units.
- Inputs will be assumed to be available everywhere (unlike in our Weber model): they are *ubiquitous*.
- Production will be according to fixed factor proportions and constant returns to scale (just as in our Weber model).
- Each firm acts on the assumption that if it changes its production decision (how much to produce) this will not change either the price it receives for its product in the city, or the price it must pay for inputs. In the jargon, each firm is a *price-taker* in the input and output markets.
Because we are assuming ubiquitous inputs, direction is also irrelevant.

If we know that a certain industry is located between $a$ and $b$ in one direction from the city, we know that it will locate between $a$ and $b$ in all directions.

This gives rise to von Thunen’s ring pattern of industrial location.

It also means that we can do our analysis on a line, in an arbitrary direction.
We can now pull all these strands together. Consider a representative firm in the $j$-th industrial sector.

- The output of this firm is sold at a fixed price $p_j$ in the city.
- Suppose that land devoted to this product (industry) has a yield (productivity) of $\alpha_j$ tons per acre.
- Production costs, exclusive of land and transportation costs, but including the next-best attainable profit, are $\beta_j$ per ton.
- Land at distance $s$ from the city costs (rents for) $R(s)$ per acre.
We can now figure out (economic) profits per acre in industry $j$ occupying land at distance $s$ from the city.

- Total revenues per acre will be the yield $\alpha_j$ per acre (tons of stuff per acre) multiplied by the market price per ton.
- So total revenues per acre in sector $j$ will be $\alpha_j p_j$
- Total economic costs per acre will be the sum of
  - Direct production costs (including opportunity costs): $\beta_j \alpha_j$
  - Transport costs: $\alpha_j ks$
  - Land costs: $R(s)$
- So total economic profits per acre, if industry $j$ locates $s$ miles from the city, will be:

$$\Pi_j = p_j \alpha_j - (\beta_j \alpha_j + \alpha_j ks + R(s))$$
We have said that land at distance $s$ goes to the industry that is prepared to bid most for it.

So our crucial question is: how much is industry $j$ prepared to bid for a plot of land at distance $s$?

We will refer to this highest possible bid for land as industry $j$’s *Bid-Rent for land (at $s$)*.
So our question is: how to determine industry j’s bid-rent for land at distance s from the city?

But this is now easy to answer: in the long run, the most that each firm in an industry could bid for land is the amount consistent with zero (economic) profits in the long run.

So we can find that bid by setting economic profits per acre equal to zero and then solving for the maximum bid for land (the land rent) that is consistent with zero economic profits.
Bid Rent in Industry \( j \) (I)

Let’s do that. We have

\[
\Pi_j = p_j \alpha_j - (\beta_j \alpha_j + \alpha_j ks + R(s))
\]

We now set this to zero, to be consistent with long-run industry equilibrium:

\[
0 = p_j \alpha_j - (\beta_j \alpha_j + \alpha_j ks + R(s))
\]

and solve for the bid-rent \( R(s) \): expand the term in brackets:

\[
0 = p_j \alpha_j - \beta_j \alpha_j - \alpha_j ks - R(s)
\]

and solve for \( R(s) \), which in this context we’ll call \( R^*_j(s) \), to remind us that is the maximum land rent (at \( s \)) that industry \( j \) can pay, consistent with long-run (zero economic profits) industry equilibrium:

\[
R^*_j(s) = p_j \alpha_j - \beta_j \alpha_j - \alpha_j ks
\]
The bid-rent function for industry $j$ is:

$$R_j^*(s) = p_j \alpha_j - \beta_j \alpha_j - \alpha_j k s$$

How can we make sense of this?

- The trick is to focus on what we want to know.
- In this case, our question is, how does this bid rent vary over space (distance).
- So let’s focus on the role of distance ($s$) in $R_j^*(s)$.
- Collecting terms, we see that the bid-rent can be written as

$$R_j^*(s) = (p_j \alpha_j - \beta_j \alpha_j) - (\alpha_j k) s$$
So the maximum that a firm in industry \( j \) can pay for land (its bid-rent) at distance \( s \) from the city is

\[
R_j^*(s) = (p_j \alpha_j - \beta_j \alpha_j) - (\alpha_j k) s
\]

Let’s simplify (clean up) this expression a bit. We’ll write, purely as notation:

\[
f_j = p_j \alpha_j - \beta_j \alpha_j \\
g_j = \alpha_j k
\]

so that \( f_j \) is profits per acre excluding transport and land costs, and \( g_j \) is transport cost per acre per mile shipped.

Then we see that

\[
R_j^*(s) = f_j - g_j s
\]
Can we visualize this expression?

- Yes: clearly it is the equation of a straight-line in distance.
- At distance $s = 0$ we have $R_j^*(s) = f_j$. If this industry is to be viable, then clearly we must have $f_j > 0$ (check the definition of $f_j$ from the previous slide).
- The slope of the straight line is $-g_j$. Since both the yield per acre in industry $j$ ($\alpha_j$) and the transport cost per ton-mile ($k$) are clearly positive, we conclude that $g_j = \alpha_j k$ is positive, and hence $-g_j$ is negative.
Bid Rent in Industry $j$ (V)

- Bid Rent in industry $j$ slopes downward in distance from the city.
- The further away from the city the industry is located, the less it is willing to bid for land.
Industrial Land Use (I)

We are now in a position to study industrial land use over space.

- Each industry \( j \) will generate a bid-rent \( R^*_j(s) \) for land at each distance \( s \) from the city.
- That bid rent represents the most that it is prepared to bid for land at distance \( s \), consistent with long-run equilibrium (entry and exit drive economic profits to zero).
- We have said that land goes to the highest bidder.
- So at any distance \( s \) the land goes to that industry whose \( R^*_j(s) \) is greatest.
Suppose we have two industries competing for land outside the city.

Their assumed bid-rents are shown as $R_1^*(s)$ and $R_2^*(s)$.

As drawn, industry 1 is prepared to bid more for land near the city.

So it locates near the city.

Industry 2 locates further out.

The point $s^*$ marks the boundary between the two industries (land uses).
For this two-industry setting, the observed pattern of land-rents over space will be the upper envelope of the individual bid-rents, shown by the heavy line.

Note that it is also downward-sloping over space, though it is not a smoothly falling line.
For more than 2 possible industries, the analysis is exactly the same.

Each industry has a linear downward-sloping bid-rent function.

At any point in space, the industry with the highest bid-rent gets the land.

This is the upper envelope (heavy line) of the bid-rents of the various industries.
Note what can happen: technological or economic conditions can result in industries being “crowded out” of the land market entirely.

This is what happens in the figure to industry 4.

It is never able to outbid the other industries.

So it is not present in the land-use pattern around this city.
What happens if transportation costs decrease?

Recall that $R_j^*(s) = f_j - g_j s$. The term $f_j$ does not involve the transport cost $k$, and only the slope $g_j = \alpha_j k$ does.

So if transport costs decline $g_j$ falls, i.e., the slope of the bid-rent function gets less steep.

Since $f_j$ is unaffected, the decrease in transportation cost causes Industry $j$’s bid-rent to pivot outward around $f_j$. 
The figure shows what happens in the 2-industry setting when transport costs decrease. Compare this to the figure on slide 32. Each industry occupies more land: the crossover-point moves outwards from the city.
Example (I)

Let's try to see how this plays out in a numerical example. Suppose we have 3 industries competing for space on our von Thunen plain around the city.

And suppose the data is:

<table>
<thead>
<tr>
<th>Industry</th>
<th>$f_j$</th>
<th>$\alpha_j$</th>
<th>$k_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Remember that $f_j$ is profit per acre excluding transport costs; $\alpha_j$ is land productivity (tons of stuff per acre) and $k_j$ is the shipping cost per acre, which we assume applies to all output.
Example (II)

Then we have, for the bid-rents:

\[
\begin{align*}
R_1(s) &= 23 - 4s \\
R_2(s) &= 20 - 3s \\
R_3(s) &= 18 - 2s
\end{align*}
\]
Example (III)

- Which industry locates closest to the city (i.e., at $s = 0$)? Clearly the one that can bid most of land there.
- So set $s = 0$ in the bid-rents, and find that industry 1 locates at the city.
- What happens next? The simplest way to see what’s going on is to draw or plot the bid rents.
- If you do this, you’ll see that industry 2 can never out-bid the other two industries:
  - between $s = 0$ and $s = 2$ it bids more than industry 3, but is out-bid by industry 1
  - beyond $s = 2$ it is out-bid by industry 3
- So land will be used by industry 1 from $s = 0$ to $s = 2\frac{1}{2}$ and by industry 3 after $s = 2\frac{1}{2}$
- When will land return to agricultural use? Clearly, when industry 3 no longer wants it, i.e., when its bid is $R_3(s) = 0$. This happens at $s = 9$. 
Example (IV)

- It may be a bit implausible to have industry 3 extend all the way from \( s = 2\frac{1}{2} \) to \( s = 9 \).
- Clearly, this is because we’re only considering three industries. If there were more, then in all likelihood another industry would outbid industry 3 at some point, and take over the land use.
- Another approach is to say that land not used by these three industries has a residual value, say \( R_A \). Then industry 3 would extend out from (in this case) \( s = 2\frac{1}{2} \) until its bid was exactly \( R_A \), after which land would be used for whatever the residual stood for.
- For example, if \( R_A = 9 \), then industry 3 would cease to be the high bidder for land when \( 18 - 2s = 9 \), i.e., when \( s = 4\frac{1}{2} \).
How can planners’ actions change this land-use pattern? Suppose that the planners do something that changes all transport costs to $k = 0.7$. Remember that in the Weber case this sort of universal change had no effect on location decisions.

But here there is an effect. Industry 2 is still crowded out, but now the switch-over point between industries 1 and 3 is

$$R_1(s) = R_3(s)$$

$$23 - (4 \times 0.7)s = 18 - (2 \times 0.7)s$$

and the solution is: $s^* = 3.5714$. The planner has succeeded in having more land closer to the city devoted to industry 1. This might be important if, for example, there are pollution problems with industry 3, and we want to keep it away from a population center.
Example (VI)

- Can we do something that will allow (induce) industry 2 to locate in our region?
- One possibility is to subsidize its transportation. This could be possible, for example, if it uses a specialized transport mode.
- So suppose transport costs per mile are 1 for industries 1 and 3, but we do something that reduces transport costs per mile to 0.85 for industry 2 only. Industry 2’s bid rent is now
  \[ R_2(s) = 20 - (3 \times 0.85)s = 20 - 2.55s. \]
- The result is that we now have the land-use pattern:
  - Industry 1: between 0 and 2.069
  - Industry 2: between 2.069 and 3.6364
  - Industry 3: from 3.6364 to 9 (assuming that the residual land value is \( R_A = 0 \)).
Another possibility, which would probably be easier to implement, is to offer industry 2 a subsidy or rebate (say on taxes) that form part of its non-transport costs.

Remembering that \( f_2 = p_2 \alpha_2 - \beta_2 \alpha_2 \) is profit per acre before transport and land costs, this could be done by changing \( \beta_2 \).

So suppose we do that, so that \( f_2 \) becomes 21 (up from 20), and industry 2’s bid rent becomes \( R_2(s) = 21 - 3s \).

The result is the new land-use pattern:

- Industry 1: between 0 and 2
- Industry 2: between 2 and 3
- Industry 3: between 3 and 9 (with \( R_A = 0 \)).
In 1925 E.W. Burgess published a von Thunen-type model of urban form. His results were that land uses were arranged in rings around a CBD: moving outwards in space from the city center, he had:

1. Downtown
2. Industrial + warehousing
3. Transition (mixed industrial + residential use)
4. Blue collar residential
5. White collar residential
6. Executive residential
The peculiar result, where the industrial + warehousing sector was located close to the downtown was based on 1920s transport technology.

The truck has barely entered the transportation system, and industry relied almost completely on railway transportation.

Railroads tended to group their facilities in centrally located switching yards to take advantage of economies of scale; in Burgess’ case, industrial location just followed the transport technology.

Nowadays, with more decentralized transport, industrial locations are more decentralized, too.
If you’re comfortable with mathematical reasoning, there’s an interesting paper in this vein that you might want to look at: