Industrial Land Use — the von Thunen Model

Philip A. Viton

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In 1826, Johann von Thunen, in *Der isolierte Stadt (The isolated city)* considered the locations of agricultural activities around an isolated city with no interregional trade.

This reflected the setting for many early-19th-century German cities, before the development of an extensive railway network.

All agricultural output needed to be transported (by horse and cart) to the city to be sold.

von Thunen’s work contained not only a sophisticated theoretical model but also a careful empirical analysis of land productivity, carrying capacity (horsepower) of the available transport, and on the need to carry additional weight (fuel) to feed the horses.
Rough pattern of von Thunen’s results for 19-th century Germany:

1. Perishables
   (fruits/vegetables/milk)
2. Lumber (heavy: you want to minimize transport costs)
3. Other crops
4. Livestock, which are their own (weight-losing) transport

(von Thunen’s actual results were more detailed than this).
Of course, von Thunen’s model was quite specialized, both as to its time and its technology.

Can we use it to get insight into contemporary land use?

Let’s try to build on the spirit of von Thunen’s model.
We will assume that all activity takes place on a von Thunen plain: a flat featureless plain.

Geographical features (topography, etc) are ignored.

We assume that there is a city located in the middle of this plain.

We will focus on land use outside the city, so we represent the city as a point in space.
Our story is about *competition for land* by the representatives of the various industrial sectors.

At any distance from the city, land could be occupied by any of the sectors.

Which sector will *actually* get land at that distance?

We will make a natural assumption here: *land goes to the highest bidder*, ie whoever is prepared to pay most for it.
So we need to ask: how much is a representative of each industry willing to pay (bid) for a plot of land located \( s \) miles from the city?

To discuss this, we need to say something about why the firms do what they do, ie their behavioral motivation.

Our answer here is conventional: firms make their decisions in order to maximize profits.

Thus, the amount a firm would be willing to bid for land at some distance from the city will depend on the level of its profits.

So we begin with a brief discussion of how profits are measured.
Accounting vs Economic Profits (I)

We will distinguish two ways of measuring profits.

- **Accounting Profits**: The definition of accounting profit is exactly what you would expect: accounting profits are the difference between total firm revenues, and the costs incurred to realize those revenues:

  \[
  \text{Accounting Profits} = \text{Total Revenues} - \text{Total Costs}
  \]

- **Economic Profits**: Economic profits take accounting profits and subtract out the profits that *could be* achieved, if resources were deployed in their next-best possible way. So

  \[
  \text{Economic Profits} = \text{Accounting Profits} - \text{Next-Best Profits}
  \]
Before we see why this is a useful distinction, let’s look at an example. Suppose there are two opportunities, called A and B, described as follows:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Revenues</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>Total Costs</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>Accounting Profit</td>
<td>18</td>
<td>25</td>
</tr>
</tbody>
</table>

Accounting profit is just the difference between total revenues and total costs, so $30 - 12 = 18$ for opportunity A, and $50 - 25 = 25$ for opportunity B.
To calculate A’s economic profit, we note that if resources devoted to opportunity A had been devoted instead to opportunity B, they would have yielded an accounting profit of 25. And since there are only two opportunities, this is the next-best profit for A.

To arrive at the economic profit for A we subtract this next-best profit (25) from A’s accounting profit: the result is $18 - 25 = -7$.

Similarly for B: its next-best opportunity is the profits offered by A, so economic profit is $25 - 18 = 7$.

The upshot is:

<table>
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<td>-7</td>
<td>7</td>
</tr>
</tbody>
</table>
What does this tell us? The important thing here is the sign of the economic profits.

- Opportunity A has a negative economic profit. This tells us that if we’re out to maximize our returns, that we can do better (we should shift resources to opportunity B).
- On the other hand, opportunity B has a positive economic profit. This tells us that there is no better way on offer to make profits (the next-best opportunity yields less profits).
We now apply these ideas to our industrial land-use problem.

- We think of an industry as made up of many small identical firms.
- In the jargon, we will consider *competitive* industries.
- Therefore we can consider a representative firm in each industry (industrial sector).
- Each industry manufactures a single product, and sells that product only in the city.
We have said that the willingness of firms to bid for land depends on their profit levels.

But what will those profit levels be?

The key to understanding this is that if a sector has positive economic profits, entrepreneurs will enter that sector; if economic profits are negative, then entrepreneurs will leave that sector (and migrate to another).
Entry and Exit (II)

- We will assume that nothing prevents a firm from entering a profitable sector, or leaving an unprofitable one.
- In the jargon, there are no *barriers to entry* (or exit) for any industrial sector.
- The most important barrier to entry found in practice is regulatory: some governmental agency restricts the ability of firms to enter (typically this question does not arise for leaving) an industrial sector.
- Examples: in many cities, entry to the taxicab sector is regulated; before the Air Deregulation Act of 1976, entry into the passenger air transport market was regulated (by the Civil Aeronautics Board).
- Some industries also experience barriers to entry in the form of very high start-up costs (think of the costs of electricity generation, or of constructing a subway system) that discourage entry.
- We are assuming that these do not apply, ie that there is Free Entry and Free Exit for all the industries that might locate on the plain.
Long-Run Profits

- If a sector has positive economic profits, entrepreneurs will be induced to enter the sector. This will reduce the profitability of the sector (more competition).
- If the sector has negative economic profits, entrepreneurs will leave the sector. This will enhance the profitability of the remaining firms (less competition).
- Our assumption of free entry and exit means that there are no reasons for these changes not to occur.
- The result is that entry and exit will eventually (in the long run) drive economic profits (but not necessarily accounting profits) to zero.
- This zero-profit outcome is the long-run industry equilibrium for a competitive industry.
- We will focus only on the long run. We will not be discussing the path to equilibrium, i.e., how we actually get there.
Firms in an Industry (I)

Let’s return now to our discussion of economic activity on a von Thunen plain.

- All inputs and output will be measured in weight units.
- Inputs will be assumed to be available everywhere (unlike in our Weber model): they are *ubiquitous*.
- Production will be according to fixed factor proportions and constant returns to scale (just as in our Weber model).
- Each firm acts on the assumption that if it changes its production decision (how much to produce), this will not change either the price it receives for its product in the city, or the price it must pay for inputs. In the jargon, it is a *price-taker* in the input and output markets.
Because we are assuming ubiquitous inputs, *direction* is also irrelevant.

If we know that an industry is located between $a$ and $b$ in one direction from the city, we know that it will locate between $a$ and $b$ in all directions.

This gives rise to von Thunen’s ring pattern of industrial location.
The Transport Sector

- We will assume that transportation to and from the city is available everywhere.
- Because direction is irrelevant, we can consider a 1-dimensional spatial structure, as if the city and its hinterland were on a line.
- We will measure distances in any direction from the city as $s$.
- Transport costs will be $k$ per ton-mile.
We can now pull all these strands together. Consider a representative firm in the $j$-th industrial sector.

- The output of this firm is sold at a fixed price per ton $p_j$ in the city.
- Land devoted to this product (industry) has a yield (productivity) of $\alpha_j$ tons per acre.
- Production costs, exclusive of land and transportation costs, but including the next-best attainable profit, are $\beta_j$ per acre. There are constant returns to scale.
- Land at distance $s$ from the city costs (rents for) $R(s)$ per acre.
We can now figure out (economic) profits per acre in industry $j$ occupying land at distance $s$ from the city.

- Total revenues per acre will be the price per ton ($p_j$) multiplied by the yield $\alpha_j$ per acre (tons per acre).
- So total revenues per acre in sector $j$ will be $\alpha_j p_j$
- Total economic costs per acre will be the sum of
  - Direct production costs (including next-best profits): $\beta_j$
  - Transport costs: $\alpha_j ks$
  - Land costs at $s$: $R(s)$
- So total economic profits per acre, if industry $j$ locates $s$ miles from the city, will be:

$$\Pi_j(s) = p_j \alpha_j - (\beta_j + \alpha_j ks + R(s))$$
We have said that land at distance $s$ goes to the industry that is prepared to bid most for it.

So our crucial question is: how much is industry $j$ prepared to bid for a plot of land at distance $s$?

We will refer to this highest possible bid for land as industry $j$’s Bid-Rent for land (at $s$).
So our question is: how to determine industry j’s bid-rent for land at distance s from the city?

But this is now easy to answer: in the long run, given free entry, the most that each firm in an industry could bid for land is the amount consistent with zero economic profits in the long run.

So we can find that bid by setting economic profits equal to zero and then solving for the maximum bid for land (land rent) that is consistent with zero economic profits.
Let’s do that. We have

$$\Pi_j = p_j \alpha_j - (\beta_j + \alpha_j ks + R(s))$$

We now set this to zero, to be consistent with long-run industry equilibrium:

$$0 = p_j \alpha_j - (\beta_j + \alpha_j ks + R(s))$$

and solve for the rent $R(s)$ which we’ll call $R_j^*(s)$, to remind us that is the maximum land rent (at $s$) that industry $j$ can pay, consistent with long-run (zero economic profits) industry equilibrium. So:

$$R_j^*(s) = p_j \alpha_j - \beta_j - \alpha_j ks$$

This is called industry $j$’s bid-rent at distance $s$ from the city.
The maximum that a firm in industry $j$ can pay for land (its bid-rent) at distance $s$ from the city is

$$R_j^*(s) = (p_j \alpha_j - \beta_j) - (\alpha_j k)s$$

Let’s clean this up, and write, purely as notation:

$$c_j = p_j \alpha_j - \beta_j$$
$$g_j = \alpha_j k$$

Then we see that industry $j$’s bid-rent can be written as:

$$R_j^*(s) = c_j - g_j s$$
How does industry $j$’s bid-rent vary over space (ie, with distance)?

- Clearly it is linear in distance.
- At distance $s = 0$ we have $R_j^*(s) = c_j$. If this industry is to be viable, then clearly we must have $c_j > 0$ (check the definition of $c_j$ from the previous slide).
- The slope of the line is $-g_j$. Since both the yield per acre in industry $j$ ($\alpha_j$) and the transport cost per ton-mile ($k$) are positive, we conclude that $g_j = \alpha_j k$ is positive, and hence $-g_j$ is negative.
Bid Rent in Industry $j$ (IV)

- Bid Rent in industry $j$ slopes downward in distance from the city.
- The further away from the city the industry is located, the less it is willing to bid for land.
Industrial Land Use (I)

We are now in a position to study industrial land use over space.

- Each industry $j$ will generate a bid-rent $R_j^*(s)$ for land at each distance $s$ from the city.
- That bid rent represents the most that it is prepared to bid for land at distance $s$, consistent with long-run equilibrium.
- We have said that land goes to the highest bidder.
- So at any distance $s$ the land goes to that industry whose $R_j^*(s)$ is greatest.
- We can describe this as saying that the actual (observed) pattern of land rents over space is the upper envelope of the individual bid-rents.
Suppose we have two industries competing for land outside the city.

- Their bid-rents are shown as $R_1^*(s)$ and $R_2^*(s)$.
- As drawn, industry 1 is prepared to bid more for land near the city.
- So it locates near the city.
- Industry 2 locates further out.
- The point $s^*$ marks the boundary between the two industries (land uses).
For this two-industry setting, the observed pattern of land-rents over space will be the upper envelope of the individual bid-rents, shown by the heavy line.

Note that it is also downward-sloping over space, though it is not a smoothly falling line (there is a kink at $s^*$).
For more than 2 possible industries, the analysis is exactly the same.

Each industry has a linear downward-sloping bid-rent function.

At any point in space, the industry with the highest bid-rent gets the land.

This is the upper envelope (heavy line) of the bid-rents of the various industries.
Note what can happen: technological or economic conditions can result in industries being “crowded out” of the land market entirely.

This is what happens in the figure to industry 4.

It is never able to outbid the other industries.

So it is not present in the land-use pattern around this city.
What happens if transportation costs decrease?
Recall that $R_j^*(s) = c_j - g_js$. The term $c_j$ does not involve the transport cost $k$, and only the slope $g_j = \alpha_j k$ does.
So if transport costs decline, $g_j$ falls, ie the slope of the bid-rent function gets less steep.
Since $c_j$ is unaffected, the decrease in transportation cost causes industry $j$’s bid-rent to pivot outward around $c_j$. 
The figure shows what happens in the 2-industry setting when transport costs decrease. Bid-rents pivot outwards. Each industry occupies more land: the crossover-point moves outwards from the city. Compare this to the figure on slide 29.
Agglomeration

- Agglomeration refers to the tendency of certain industries to cluster together.
- Let’s try to construct a simple model of this.
- As before, we’ll concentrate in a representative firm in each industry.
- We need a reason for industries to cluster — they have to have something to do with one another.
- Let’s restrict our attention to a two-industry model, and we’ll suppose that the output of industry 2 is used as an input to production in industry 1. That will be its only use: industry 2’s output will not be sold at the city at all.
- So the reason to cluster, if it occurs, is to be near your customer (for industry 2) or your input supplier (industry 1).
Industry 1

Industry 1 will be as before, except that (a) every ton of output requires 1 ton of the output of industry 2; and (b) we will let industry 1 have its own transportation cost per ton-mile, $k_1$. Note that because all of 1’s output is sold at the city, we don’t need to worry about its price varying over space and we write it as $p_1$.

But industry 2’s output is sold wherever industry 1 locates, so its price will vary with space. If $p_2^*(s)$ is the selling price per ton of good 2 at location $s$, then industry 1’s cost per acre from 2’s input is $p_2^*(s)\alpha_1$, where as before $\alpha_1$ is the productivity of land for industry 1.

So industry 1’s profit per acre at $s$ is

$$\Pi_1(s) = p_1\alpha_1 - (\beta_1 + k_1\alpha_1 s + R_1(s) + p_2^*(s)\alpha_1)$$

And, imposing the zero (economic) profits condition, we have, for its bid-rent at $s$:

$$R_1^*(s) = p_1\alpha_1 - \beta_1 - p_2^*(s)\alpha_1 - k_1\alpha_1 s$$
Industry 2

Assume that agglomeration does actually take place.

Then industry 2 will have no transport costs.

So industry 2’s profit per acre at $s$ is:

$$\Pi_2(s) = p_2^*(s)\alpha_2 - (\beta_2 + R_2(s))$$

where as before $\alpha_2$ is industry 2’s productivity per acre, and $\beta_2$ is its other costs per acre.

Then under free entry, we can find industry 2’s bid-rent by equating profits per acre to zero. The result is:

$$R_2^*(s) = p_2^*(s)\alpha_2 - \beta_2$$
Conditions for Agglomeration (I)

When will the two industries locate in the same place? There are two conditions:

1. They must offer the same bid-rent at $s$.
2. It must be unprofitable for industry 2 to ship its product anywhere.
Conditions for Agglomeration

As to the first condition, we equate the bid rents of the two industries and find

\[ R_1^*(s) = R_2^*(s) \]
\[ p_1 \alpha_1 - \beta_1 - p_2^*(s) \alpha_1 - k_1 \alpha_1 s = p_2^*(s) \alpha_2 - \beta_2 \]

which gives us an expression for the equilibrium price for industry 2's output at any location:

\[ p_1 \alpha_1 - \beta_1 - k_1 \alpha_1 s + \beta_2 = p_2^*(s) \alpha_2 + p_2^*(s) \alpha_1 \]
\[ p_1 \alpha_1 - \beta_1 - k_1 \alpha_1 s + \beta_2 = p_2^*(s) (\alpha_1 + \alpha_2) \]
\[ p_2^*(s) = \frac{p_1 \alpha_1 - \beta_1 - k_1 \alpha_1 s + \beta_2}{(\alpha_1 + \alpha_2)} \]
As to the second condition, it must be unprofitable for industry 2 to ship any product. Now, $p_2^*(s)$ is the price industry 2 receives per ton, i.e., its total revenue per ton at $s$, which we can write as

$$p_2^*(s) = \frac{p_1 \alpha_1 - \beta_1 + \beta_2}{\alpha_1 + \alpha_2} - \frac{k_1 \alpha_1}{\alpha_1 + \alpha_2} s$$

If industry 2 were to ship 1 ton of output 1 mile, the change in its revenues per ton is

$$-\frac{k_1 \alpha_1}{\alpha_1 + \alpha_2}$$

multiplied by whether we add (+1) or subtract (−1) from shipments.

If $k_2$ is the shipping cost per ton-mile, the shipment will add $k_2$ to costs.
Conditions for Agglomeration (III)

- In an agglomeration equilibrium, a change in any direction must be unprofitable, so industry 2’s transportation cost $k_2$ per ton must be greater than this, so

$$k_2 \geq \left\lfloor \frac{k_1 \alpha_1}{(\alpha_1 + \alpha_2)} \right\rfloor$$

- Then we have, remembering that everything is positive,

$$\frac{k_2}{k_1} \geq \frac{\alpha_1}{(\alpha_1 + \alpha_2)}$$

- The ratio of the transportation costs per ton-mile in the two industries must exceed the proportional productivity of the two industries.

- When this condition is not satisfied, then the two industries will locate at different point in space (just like our first model), except that industry 2 will ship product only to industry 1’s location.
In 1925 E.W. Burgess published a von Thunen-type model of urban form.

His results were that land uses were arranged in rings around a CBD: moving outwards in space from the city center, he had:

1. Downtown
2. Industrial + warehousing
3. Transition (mixed industrial + residential use)
4. Blue collar residential
5. White collar residential
6. Executive residential
The peculiar result, where the industrial + warehousing sector was located close to the downtown was based on 1920s transport technology.

The truck has barely entered the transportation system, and industry relied almost completely on railway transportation.

Railroads tended to group their facilities in centrally located switching yards to take advantage of economies of scale; in Burgess’ case, industrial location just followed the transport technology.

Nowadays, with more decentralized transport, industrial locations are more decentralized, too.
If you’re comfortable with mathematical reasoning, there’s in interesting paper in this vein that you might want to look at:


The model of agglomeration is a light re-write of a model in chapter 3 of the advanced book: