Producer’s Surplus

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Introduction

- We have seen how to evaluate consumer’s wtp for a market-induced price change: we measure this (with appropriate cautions, primarily that we treat it as an approximation) by the change in consumer’s surplus, the area under the demand function between the two price horizontals.
- We now address to the remaining question in market-induced (price) changes: how to evaluate the benefits to producers or firms.
- Fortunately this is easy, and has none of the complications of consumer’s surplus.
- The obviously correct measure of benefits to firms is profits.
- If a project changes the level of the firm’s profits, we measure the impact by the change in its profits.

Profits

- Consider a firm selling $x_1$ units of output at price $p_1$.
- Its Total Revenues (TR) are $p_1 \cdot x_1$.
- Write its Total Costs as $TC$.
- Then profits are Total Revenues minus Total Costs or:

$$\Pi = TR - TC.$$

Total Costs: Fixed and Variable

- Typically a firm will have both fixed ($TFC$) and variable ($TVC$) costs.
- So re-write total costs as:

$$TC = TFC + TVC$$

- Then we have:

$$\Pi = TR - (TFC + TVC)$$
Suppose a project changes revenues from $TR^a$ to $TR^b$ and costs from $TC^a$ to $TC^b$.

Using our decomposition of costs, we have:

\[
\Delta \Pi = \Pi^b - \Pi^a = (TR^b - TC^b) - (TR^a - TVC^a) = (TR^b - (TFC^b + TVC^b)) - (TR^a - (TFC^a + TVC^a)) = (TR^b - TVC^b) - (TR^a - TVC^a)
\]

since by definition fixed costs are unchanged by the project.

We have:

\[
\Delta \Pi = (TR^b - TVC^b) - (TR^a - TVC^a)
\]

The quantity

\[
PS = TR - TVC
\]

is called the firm’s *Producer Surplus*.

So the change in the firm’s profits ($\Delta \Pi$) is the same as the change in producer’s surplus:

\[
\Delta \Pi = PS^b - PS^a = \Delta PS
\]

It is important to understand that while the *change* in profits equals the *change* in producer’s surplus, profits and producer’s surplus themselves are not the same.

The two differ by the amount of fixed costs:

\[
\Pi = TR - (TVC + TFC) = TR - TVC - TFC = PS - TFC
\]

But since for project evaluation we will always be looking at the *change* in producer’s surplus, this won’t matter.

Finally, remember that unlike the case of consumer’s surplus, producer’s surplus isn’t an approximation to anything more theoretically appropriate.

Using the change in producer’s surplus ($\Delta PS$) has two advantages over the change in profits ($\Delta \Pi$):

1. It does not require us to understand fixed costs; and empirically, fixed costs (which will be the value of the constant or intercept in a cost-function regression equation) is often imprecisely estimated.

2. If we know something about the firm’s marginal cost schedule, it is easy to calculate total variable costs: they are simply the area under the marginal cost curve from $x_1 = 0$ (zero output) to $x_1^a$ (observed output). See the next slide.
Total Variable Costs

- The figure shows a firm’s upward-sloping marginal cost curve.
- If it is a profit-maximizing price-taker, then, if it faces market price $p^a$, its output will be $x^a_1$.
- Total revenue is $p^a_1 \cdot x^a_1$ (hatched).
- Total variable costs are the area under MC from $x_1 = 0$ to $x_2 = x^a_1$ (dark shade).
- So producer’s surplus is the area above MC but below the price horizontal (light shade).

Project

- Suppose the firm is a price-taker in the output market, and that our project has the effect of raising the firm’s market price to $p^b_1$. The firm responds by raising its output to $x^b_1$.
- The new total revenue is the hatched area, total variable costs are darkly shaded and producer’s surplus is lightly shaded.

Change in Producer’s Surplus

- Comparing the previous pictures, we can see that the change in producer’s surplus is the area to the left of the marginal cost curve and between the two price horizontals.
- Note that this is valid only for a competitive firm that sees its market price change with no shift in its cost curves.

Multiproduct Producer’s Surplus

- When a firm produces several products and the project changes their prices, we naturally measure the full impact as the change in producer’s surplus in each of the product markets. So:

$$\Delta PS = \Delta PS_1 + \Delta PS_2 + \ldots$$

(just as we did for consumer’s surplus).
- It can be shown that $\Delta PS$ is not path-dependent: that is, for a multiproduct firm you will get the same answer in whatever order you do the calculation (unlike consumer’s surplus).
Market Impacts — Full Analysis

We are now in a position to evaluate the full impact of a policy that changes a market price from its pre-project level \( p^a \) to its post project level \( p^b \).

- For the impact on consumers we use the change in consumer’s surplus:
  \[ \Delta CS = CS(p^b) - CS(p^a) \]
- For the impact on firms we use the change in producer’s surplus (aggregating over firms, if necessary):
  \[ \Delta PS = PS(p^b) - PS(p^a) \]
- And then the full impact is:
  \[ \Delta W = \Delta CS + \Delta PS \]

Example — Monopolization

As an example of how all this fits together, let’s reconsider a policy that allows an industry to become monopolized. So our pre-project state (a) is competition, and our post-project state (b) is monopolization. Note that we are assuming that this is the only change in the economy.

- We saw last quarter that monopolization has two impacts: it reduces the quantity produced and increases the price consumers pay for the product.
- Intuitively, this is a bad thing; but not so fast. We would naturally assume that the price increase leads to an increase in profits: and might not this gain outweigh the loss to consumers?
- By using our new tools, we can see that this is not so. Monopolization is an overall net “bad” to the economy.

Monopolization — Setup

- The figure shows the basic position: under competition, demand = supply, where supply is the sum of the firms’ marginal costs; the result is price \( p^a \) and output \( x^a \).
- Under monopolization, profit-maximizing output is governed by the condition \( MC = MR \), leading to output \( x^b \). The monopolist prices to sell all output, resulting in a price \( p^b \).

Monopolization – Impact on Consumers

- As usual, the impact on consumers is \( \Delta CS = \) signed area under the demand function between the two price horizontals.
- This is the shaded area, and it is important to note that it is a negative quantity.
Producers’ Surplus under Competition

- Pre-project producer’s surplus $PS^a$ is $TR ( = p^a x^a) - TVC$ (= area under MC from $x = 0$ to $x = x^a$).
- In the figure, $PS^a$ is shaded.

Producers’ Surplus under Monopoly

- Similarly, relative to the post-project monopoly equilibrium $(x^b, p^b)$ post-project producer’s surplus $PS^b$ is the shaded area.

Change in Producer’s Surplus

- Comparing the two previous figures, $\Delta PS$ is the shaded area
- Note that this has two parts: a rectangle (c), representing where $PS^b$ is more than $PS^a$ and hence has a positive sign; and a triangle (d) representing part of $PS^a$ not included in $PS^b$; so this has a negative sign.
- Obviously the total effect is positive: as we’d expect, monopoly increases firm profits.

Net Impact I

- For the overall net impact, we compare $\Delta PS$ to $\Delta CS$.
- First, area $a$ in the consumer’s surplus picture exactly matches area $c$ in the producer’s surplus picture. The signs are opposite ($a$ is negative, $c$ is positive) so they cancel.
**Net Impact II**

- Second, area $b$ from the consumer's surplus picture is not included in the producer's surplus picture. So we need to include it here.
- Third, area $d$ from the producer's surplus picture is not included in the consumer's surplus picture. So we need to include that, too.

**Net Impact III**

- The result is that the overall change in welfare $\Delta W$ is the shaded area.
- Note that both portions of this are negative quantities.
- So our result is that $\Delta W < 0$ : monopolization definitely decreases social welfare.

**Conclusion**

- We conclude that going from a situation of competition to one of monopoly has an unambiguously negative impact.
- In the literature, this is referred to as the Deadweight Loss of monopolization.

**Computational Considerations I**

In the context of our monopolization example, suppose you need to calculate the pre-project equilibrium $(p^a, x^a)$. Conceptually, you want to equate supply ($= MC$) and demand.

- Note that the marginal cost function is $MC(x)$ : that is, it takes a quantity and delivers (tells us) its marginal cost, in dollars.
- But the demand function is $x^*(p)$ : that is, it takes a price and delivers the demand (quantity) at that price.
- So you can’t just equate $x^*(p)$ and $MC(x)$ : that would be meaningless.
- Instead, you must first invert (solve) either the demand function or the marginal cost function, so they both deliver the same thing (output or money). Then you can equate the two equations.
For example, suppose in the linear case you have:

- Marginal cost: \( c = \alpha_0 + \alpha_1 x \) \((\alpha_1 > 0)\)
- Demand: \( x = \beta_0 + \beta_1 p \) \((\beta_1 < 0)\)

The in order to find the equilibrium you can invert the demand function, giving \( p = (x - \beta_0) / \beta_1 \) and then equate demand and supply:

\[
\frac{(x - \beta_0)}{\beta_1} = \alpha_0 + \alpha_1 x
\]

which is an equation in quantity only.

Alternatively, you can just feed in the price (cost) equation into the demand function:

\[
x = \beta_0 + \beta_1 p = \beta_0 + \beta_1 (\alpha_0 + \alpha_1 x)
\]

which is also an equation in \( x \) only; and solve.

The solution turns out to be (assuming \( \alpha_1 \beta_1 \neq 1 \))

\[
x = \frac{\beta_0 + \alpha_0 \beta_1}{1 - \alpha_1 \beta_1}
\]

And the competitive price (= marginal cost) is

\[
p = \alpha_0 + \alpha_1 x = \alpha_0 + \alpha_1 \left( \frac{\beta_0 + \alpha_0 \beta_1}{1 - \alpha_1 \beta_1} \right)
\]