Traffic Congestion: The Mechanics

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The Problem

- Traffic moves too slowly (especially at peak times).
- Too many vehicles competing for too little road space.
- People are making too many trips (especially during peak periods).

Extent of the Problem: 2009

<table>
<thead>
<tr>
<th>Area</th>
<th>Daily VMT/Lane-Mile</th>
<th>Ann Hrs Delay/Person</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Average</td>
<td>15,391</td>
<td>25</td>
</tr>
<tr>
<td>Boston</td>
<td>15,610</td>
<td>28</td>
</tr>
<tr>
<td>Chicago</td>
<td>18,018</td>
<td>44</td>
</tr>
<tr>
<td>Dallas</td>
<td>17,199</td>
<td>32</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>23,447</td>
<td>40</td>
</tr>
<tr>
<td>New York</td>
<td>15,735</td>
<td>24</td>
</tr>
<tr>
<td>San Francisco</td>
<td>19,110</td>
<td>30</td>
</tr>
<tr>
<td>Washington DC</td>
<td>18,048</td>
<td>41</td>
</tr>
<tr>
<td>Cincinnati</td>
<td>14,385</td>
<td>13</td>
</tr>
<tr>
<td>Cleveland</td>
<td>11,793</td>
<td>13</td>
</tr>
<tr>
<td>Columbus</td>
<td>14,909</td>
<td>11</td>
</tr>
</tbody>
</table>

Note: this is all travel, not peak travel, so it probably underestimates the congestion problem.

Congestion: Average of 437 US Urban Areas

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Freeways</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMT ('000)</td>
<td>2,021</td>
<td>3,959</td>
<td>4,166</td>
<td>1.9</td>
</tr>
<tr>
<td>Lane Miles</td>
<td>243</td>
<td>279</td>
<td>299</td>
<td>1.2</td>
</tr>
<tr>
<td>VMT/Lane-Mile</td>
<td>9,058</td>
<td>14,190</td>
<td>13,933</td>
<td>1.5</td>
</tr>
<tr>
<td>Arterials</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMT ('000)</td>
<td>3,088</td>
<td>4,402</td>
<td>4,521</td>
<td>1.5</td>
</tr>
<tr>
<td>Lane Miles</td>
<td>796</td>
<td>890</td>
<td>939</td>
<td>1.2</td>
</tr>
<tr>
<td>VMT/Lane-Mile</td>
<td>3,879</td>
<td>4,944</td>
<td>4,815</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note: this is all travel, not peak travel, so it probably underestimates the congestion problem.
### Congestion in Columbus OH

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Freeways</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMT ('000)</td>
<td>6,200</td>
<td>12,000</td>
<td>14,599</td>
<td>2.4</td>
</tr>
<tr>
<td>Lane Miles</td>
<td>735</td>
<td>860</td>
<td>970</td>
<td>1.3</td>
</tr>
<tr>
<td>VMT/Lane-Mile</td>
<td>8,435</td>
<td>13,953</td>
<td>15,051</td>
<td>1.8</td>
</tr>
<tr>
<td>Arterials</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMT ('000)</td>
<td>3,010</td>
<td>9,300</td>
<td>9,909</td>
<td>3.3</td>
</tr>
<tr>
<td>Lane Miles</td>
<td>1,005</td>
<td>1,685</td>
<td>2,177</td>
<td>2.2</td>
</tr>
<tr>
<td>VMT/Lane-Mile</td>
<td>2,995</td>
<td>5,519</td>
<td>4,552</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Note: this is all travel, not peak travel, so it probably underestimates the congestion problem. Also: annexation issues.

### Some Proposed Solutions

- Build more capacity (roads)
- Smart roads
- Better maintenance (bad maintenance — eg potholes — slows down traffic)
- Flex time
- Encourage/Incentivize transit use
- Encourage/Incentivize carpooling
- Encourage/Incentivize walking
- Land-use changes

### Approach, and Preview

Our approach to traffic congestion has two parts:

1. First we develop the needed concepts (usually from traffic engineering).
2. Then we apply them to the problem of congestion.

### Engineering Concepts: Vehicle Impact

- Different vehicles will have different impacts on the traffic flow
- Engineers measure these impacts relative to a standard
- The standard is: relative to the impact of a single mid-sized passenger car
  - This is a vehicle’s *Passenger Car Equivalent* (PCE)
- PCE’s differ by road layout (geometric design) and grade
- Some typical values are:
  - Mid-sized passenger car: PCE = 1.0
  - Transit Bus: PCE ≈ 1.8
  - Large truck: PCE ≈ 2.4
- Rule of thumb: PCE ≈ [length of vehicle in feet] ÷ 14
PCEs and the Traffic Stream — Example

- Suppose a traffic stream (vehicles past a given point, per hour) consists of:
  - 5000 mid-sized passenger cars
  - 30 transit buses
  - 200 small trucks
- Then the PCEs for this stream are:
  \[ PCE = 5000 + (30 \times 1.8) + (200 \times 2.4) \]
  \[ PCE = 5534.0 \]
- In other words, this traffic stream is equivalent (for purposes of flow) to one consisting just of 5534 mid-sized passenger cars.
- Obviously this can be extended to other PCE values (vehicle types).
- For the rest of our discussion, we assume that a calculation like this has been done, and we consider a traffic stream’s volume to be in PCE units.

Volume, Concentration, Capacity

- Traffic volume \( q \) : average flow of PCE’s past a given point (PCE’s / unit time).
  \[ q = \frac{\text{Number of PCE’s past a point in } T \text{ time units}}{T} \]
  \[ q = n/T \]
- Traffic concentration (density) \( k \) : PCE’s per unit length of road (PCE’s per mile, say), at a point in time.
  \[ k = n/L \] where \( L \) is the length of the road under consideration
- Road capacity \( c \) : number of PCE’s that a given road is designed to support (PCE’s / mile / hour, say).

Speed

Engineers have two ways of thinking about the average speed on a road.

- Time-mean (average) speed
  - Average of the observed vehicle speeds
  - If we see \( n \) vehicles in a time period, and vehicle \( i \) is going at speed \( s_i \), then
  \[ \text{Time-mean speed} = \frac{1}{n} \sum s_i \]
- Space-mean (average) speed, \( u \)
  - We have: \( d = s \times t \) (distance = speed \times time) so \( s = d/t \)
  - Then
  \[ \text{Space-mean speed} u = \frac{\text{Total distance travelled in } T \text{ time units}}{\text{Total time taken to traverse that distance}} \]
- Unless otherwise stated, when engineers talk about average speeds, they mean space-mean speeds.

Measuring Space-Mean Speed

Vehicle travels \( L \) distance units in \( t_2 - t_1 \) time units. Contributes \((t_2 - t_1)/L\) to space-mean speed.
Two Measures of Speed I

- It’s important to realize that space-mean and time-mean speeds will usually differ.
- To see this, suppose we have a stretch of road 1.5 miles long, and we observe 4 vehicles during some particular observation period.

<table>
<thead>
<tr>
<th>Vehicle number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed speed (mph)</td>
<td>39</td>
<td>48</td>
<td>40</td>
<td>52</td>
</tr>
<tr>
<td>Travel time (mins)</td>
<td>2.31</td>
<td>1.875</td>
<td>2.25</td>
<td>1.73</td>
</tr>
<tr>
<td>Travel time (hours)</td>
<td>0.03846</td>
<td>0.03125</td>
<td>0.03751</td>
<td>0.02885</td>
</tr>
</tbody>
</table>

Then we have

\[
\text{time-mean speed} = \frac{39 + 48 + 40 + 52}{4} = 44.75 \text{ mph}
\]

While

\[
\text{space-mean speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{6 \text{ miles}}{0.13607 \text{ hrs}} = 44.095 \text{ mph}
\]

Fundamental Relation of Traffic Engineering

- If
  - \(q\) = average traffic volume (PCE’s)
  - \(k\) = average traffic concentration (PCE’s / unit road length)
  - \(u\) = space-mean speed
- Then the Fundamental Relation says:

\[ q = ku \]

Fundamental Diagram of Traffic Engineering
Suppose we observe concentration $k_1$

Then we can read off the average flow as $q_1$

Now consider the slope of line $0A$: this is rise $/\text{run} = q_1 / k_1$

By Fundamental Relation $q = kv$, this slope is the space-mean speed.
Level of Service II

- Note that LOS-F corresponds to the backward-bending part of the speed-versus-volume diagram

Empirical Measurement

- We turn now to empirical measures of the relation between traffic volume, road capacity, and (space-mean) speeds.
- This is usually done in terms of the volume-to-capacity ratio, $q/c$

Approach I — The BPR Function

- Developed by the US Bureau of Public Roads
- Intended to be able to describe a wide variety of roads and circumstances
- Functional form:

  \[
  \text{Travel time per mile } t = \gamma + \beta \left( \frac{q}{c} \right)^\kappa
  \]

- Where:
  - Travel time $t$ is in minutes per mile
  - $q =$ traffic flow (PCE's)
  - $c =$ road capacity (PCE's per minute/mile)
  - $\gamma$, $\beta$, $\kappa$ are parameters

BPR Function — Interpretation of Parameters

- Suppose no traffic ($q = 0$)
  - Then Travel time per mile $t = \gamma$
  - So $\gamma$ can be interpreted as the free-flow travel time per mile
- Conversely, suppose the volume-to-capacity ratio is 1: ($q/c = 1$).
  - Then Travel time per mile $t = \gamma + \beta$
  - So $\gamma + \beta$ is the travel time per mile corresponding roughly to LOS-E
- The parameter $\kappa$ is empirically estimated, and varies by type of road.
  - Observation suggests $\kappa = 4$ for an urban expressway; $\kappa = 2.5$ for an urban arterial
Consider a road on which we expect the free-flow speed to be 65 mph, and when the road is at capacity \( \frac{q}{c} = 1 \) we expect traffic to move at 25 mph.

We have 
\[
\gamma = 65 \frac{\text{mi}}{\text{hr}} = \frac{65 \text{ min}}{60 \text{ min}} = 0.92 \frac{\text{min}}{\text{mi}}
\]
And 
\[
\gamma + \beta = 25 \frac{\text{mi}}{\text{hr}} = \frac{60 \text{ min}}{25 \text{ mi}} = 2.4 \frac{\text{min}}{\text{mi}}
\]
So 
\[
\gamma + \beta = 2.4 \\
\beta = 2.4 - \gamma = 1.48
\]
Finally:
\[
\text{Travel time per mile} = 0.92 + 1.48 \left( \frac{q}{c} \right) ^\kappa
\]
To convert travel time \( t \) in mins/mile to speed in mph, use \( u = 60 / t \)

Consider a 4-lane expressway, with per-lane capacity = 2000 PCE's/hr.
- So \( c = 2000 \times 4 = 8000 \)
- Take free-flow speed = 65 mph, speed at \( q/c = 1 \) to be 25 mph
- Then \( \gamma = 0.92 ; \beta = 1.48 \) (as we've just computed)
- For an expressway, take \( \kappa = 4 \)

<table>
<thead>
<tr>
<th>( q/\text{lane} )</th>
<th>( q )</th>
<th>( q/c )</th>
<th>( t )</th>
<th>( u, \text{mph} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>2000</td>
<td>0.25</td>
<td>0.926</td>
<td>64.8</td>
</tr>
<tr>
<td>1000</td>
<td>4000</td>
<td>0.50</td>
<td>1.013</td>
<td>59.3</td>
</tr>
<tr>
<td>1500</td>
<td>6000</td>
<td>0.75</td>
<td>1.388</td>
<td>43.2</td>
</tr>
<tr>
<td>1600</td>
<td>6400</td>
<td>0.80</td>
<td>1.526</td>
<td>39.3</td>
</tr>
<tr>
<td>1700</td>
<td>6800</td>
<td>0.85</td>
<td>1.693</td>
<td>35.4</td>
</tr>
<tr>
<td>1800</td>
<td>7200</td>
<td>0.90</td>
<td>1.891</td>
<td>31.7</td>
</tr>
<tr>
<td>1900</td>
<td>7600</td>
<td>0.95</td>
<td>2.125</td>
<td>28.2</td>
</tr>
</tbody>
</table>

For a given road, we know its (design) capacity \( c \)
- We observe traffic volume \( q_t \) and the associated speeds \( u_t \) at times \( t \)
- We then fit a curve to the data \( ((q_1, u_1), (q_2, u_2), (q_3, u_3), \ldots) \)
- Note that according to our previous picture of speed versus volume, we want to fit a curve to the data, not a straight line

Roads: Several expressways in Oakland, California
- Data: collected by the California Division of Highways in 1970
- Estimation: Keeler & Small (1977)
- Assumed functional form:
\[
\frac{q_t}{c} = \alpha_0 + \alpha_1 u_t + \alpha_2 u_t^2
\]
- \( c \) is assumed known: Keeler & Small took \( c = 2000 \) (PCE’s/lane/hour)
- The \( \alpha \)'s are to be estimated. Note that this is linear-in-parameters
Statistical Estimation — Example II

- Starting from
  \[ \frac{q_t}{c} = \alpha_0 + \alpha_1 u_t + \alpha_2 u_t^2 \]
- We can invert to find a relation for speeds. In general we have
  \[ u_t = \left( -\frac{\alpha_1}{2\alpha_2} \right) \pm \sqrt{\frac{\alpha_1^2}{4\alpha_2^2} - \frac{1}{\alpha_2} \left( \alpha_0 - \frac{q_t}{c} \right)} \]
- If we ignore the possibility of LOS-F then we take the positive branch
- Some estimation results:

<table>
<thead>
<tr>
<th>Road</th>
<th>( \hat{\alpha}_0 )</th>
<th>( \hat{\alpha}_1 )</th>
<th>( \hat{\alpha}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastshore Freeway</td>
<td>-3.153</td>
<td>0.1767</td>
<td>-0.001923</td>
</tr>
<tr>
<td>Bayshore Freeway</td>
<td>-1.430</td>
<td>0.1047</td>
<td>-0.001187</td>
</tr>
<tr>
<td>Nimitz Freeway</td>
<td>-0.588</td>
<td>0.1455</td>
<td>-0.002404</td>
</tr>
</tbody>
</table>

Statistical Estimation — Example III

- For the Eastshore Freeway we have
  \[
  \begin{align*}
  \frac{q_t}{w} & = \frac{64.4}{500} = 0.1288 \\
  \frac{u_t}{c} & = \frac{64.4}{1000} = 0.0644 \\
  \frac{q_t}{w} & = \frac{64.4}{1500} = 0.0429 \\
  \frac{u_t}{c} & = \frac{64.4}{1600} = 0.0403 \\
  \frac{q_t}{w} & = \frac{64.4}{1700} = 0.0378 \\
  \frac{u_t}{c} & = \frac{64.4}{1800} = 0.0356
  \end{align*}
\]

Statistical Estimation — Example IV

- Take the case of the Eastshore Freeway.
- Then:
  \[
  \begin{align*}
  \frac{-\alpha_1}{2\alpha_2} & = \frac{-0.1767}{2 \times (-0.001923)} = 45.94 \\
  \frac{\alpha_1^2}{4\alpha_2^2} & = \frac{(-0.1767)^2}{4(-0.001923)^2} = 2110.84 \\
  \frac{1}{\alpha_2} & = \frac{1}{-0.001923} = -520.02
  \end{align*}
  \]
- And:
  \[ u_t = 45.94 + \sqrt{2110.84 + 520.02 \left( -3.153 - \frac{q_t}{c} \right)} \]
- And after a certain amount of cleaning up:
  \[ u_t = 45.94 + \sqrt{471.22 - 0.26 \frac{q_t}{w}} \]
  where we have used \( c = 2000w \), where \( w \) is the number of lanes in a given direction.
Now that we understand the mechanics, we are in a position to ask: what has gone wrong?

- Why are (peak) speeds so low, and why is there so much peak traffic?
- Why will the suggested fixes not work (at least in the long run)?

Sources