Traffic Congestion: The Mechanics

Philip A. Viton

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The Problem

- Traffic moves too slowly (especially at peak times).
- Too many vehicles competing for too little road space.
- People are making too many trips (especially during peak periods).
<table>
<thead>
<tr>
<th>Area</th>
<th>Daily VMT /Lane-Mile</th>
<th>Ann Hrs Delay /Person</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Average</td>
<td>15,391</td>
<td>25</td>
</tr>
<tr>
<td>Boston</td>
<td>15,610</td>
<td>28</td>
</tr>
<tr>
<td>Chicago</td>
<td>18,018</td>
<td>44</td>
</tr>
<tr>
<td>Dallas</td>
<td>17,199</td>
<td>32</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>23,447</td>
<td>40</td>
</tr>
<tr>
<td>New York</td>
<td>15,735</td>
<td>24</td>
</tr>
<tr>
<td>San Francisco</td>
<td>19,110</td>
<td>30</td>
</tr>
<tr>
<td>Washington DC</td>
<td>18,048</td>
<td>41</td>
</tr>
<tr>
<td>Cincinnati</td>
<td>14,385</td>
<td>13</td>
</tr>
<tr>
<td>Cleveland</td>
<td>11,793</td>
<td>13</td>
</tr>
<tr>
<td>Columbus</td>
<td>14,909</td>
<td>11</td>
</tr>
</tbody>
</table>
### Congestion: Average of 437 US Urban Areas

<table>
<thead>
<tr>
<th></th>
<th>1984</th>
<th>2000</th>
<th>2010</th>
<th>Change 1982-2010</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Freeways</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMT ('000)</td>
<td>2,021</td>
<td>3,959</td>
<td>4,166</td>
<td>1.9</td>
</tr>
<tr>
<td>Lane Miles</td>
<td>243</td>
<td>279</td>
<td>299</td>
<td>1.2</td>
</tr>
<tr>
<td>VMT/Lane-Mile</td>
<td>9,058</td>
<td>14,190</td>
<td>13,933</td>
<td>1.5</td>
</tr>
<tr>
<td><strong>Arterials</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMT ('000)</td>
<td>3,088</td>
<td>4,402</td>
<td>4,521</td>
<td>1.5</td>
</tr>
<tr>
<td>Lane Miles</td>
<td>796</td>
<td>890</td>
<td>939</td>
<td>1.2</td>
</tr>
<tr>
<td>VMT/Lane-Mile</td>
<td>3,879</td>
<td>4,944</td>
<td>4,815</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note: this is all travel, not peak travel, so it probably underestimates the congestion problem.
## Congestion in Columbus OH

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Freeways</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMT (’000)</td>
<td>6,200</td>
<td>12,000</td>
<td>14,599</td>
<td>2.4</td>
</tr>
<tr>
<td>Lane Miles</td>
<td>735</td>
<td>860</td>
<td>970</td>
<td>1.3</td>
</tr>
<tr>
<td>VMT/Lane-Mile</td>
<td>8,435</td>
<td>13,953</td>
<td>15,051</td>
<td>1.8</td>
</tr>
<tr>
<td><strong>Arterials</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMT (’000)</td>
<td>3,010</td>
<td>9,300</td>
<td>9,909</td>
<td>3.3</td>
</tr>
<tr>
<td>Lane Miles</td>
<td>1,005</td>
<td>1,685</td>
<td>2,177</td>
<td>2.2</td>
</tr>
<tr>
<td>VMT/Lane-Mile</td>
<td>2,995</td>
<td>5,519</td>
<td>4,552</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Note: this is all travel, not peak travel, so it probably underestimates the congestion problem. Also: annexation issues.
Some Proposed Solutions

- Build more capacity (roads)
- Smart roads
- Better maintenance (bad maintenance — eg potholes — slows down traffic)
- Flex time
- Encourage/Incentivize transit use
- Encourage/Incentivize carpooling
- Encourage/Incentivize walking
- Land-use changes
Our approach to traffic congestion has two parts:

1. First we develop the needed concepts (usually from traffic engineering).
2. Then we apply them to the problem of congestion.
Different vehicles will have different impacts on the traffic flow.
Engineers measure these impacts relative to a standard.
The standard is: relative to the impact of a single mid-sized passenger car.
This is a vehicle's *Passenger Car Equivalent* (PCE).
PCE's differ by road layout (geometric design) and grade.
Some typical values are:
- Mid-sized passenger car: PCE = 1.0
- Transit Bus: PCE ≈ 1.8
- Large truck: PCE ≈ 2.4

Rule of thumb: PCE ≈ [length of vehicle in feet] ÷ 14
PCEs and the Traffic Stream — Example

Suppose a traffic stream (vehicles past a given point, per hour) consists of
- 5000 mid-sized passenger cars
- 30 transit buses
- 200 small trucks

Then the PCEs for this stream are:

\[
PCE = 5000 + (30 \times 1.8) + (200 \times 2.4)
\]
\[
= 5534.0
\]

In other words, this traffic stream is equivalent (for purposes of flow) to one consisting just of 5534 mid-sized passenger cars.

Obviously this can be extended to other PCE values (vehicle types).

For the rest of our discussion, we assume that a calculation like this has been done, and we consider a traffic stream’s volume to be in PCE units.
Volume, Concentration, Capacity

- Traffic volume \( q \): average flow of PCE’s past a given point (PCE’s / unit time).
  
  \[ q = \frac{\text{Number of PCE's } n}{\text{T time units}} \]

- Traffic concentration (density) \( k \): PCE’s per unit length of road (PCE’s per mile, say), at a point in time.
  
  \[ k = \frac{n}{L} \] where \( L \) is the length of the road under consideration

- Road capacity \( c \): number of PCE’s that a given road is *designed* to support (PCE’s / mile / hour, say).
**Speed**

Engineers have two ways of thinking about the average speed on a road.

- **Time-mean (average) speed**
  - Average of the observed vehicle speeds
  - If we see $n$ vehicles in a time period, and vehicle $i$ is going at speed $s_i$, then
    \[
    \text{Time-mean speed} = \frac{1}{n} \sum s_i
    \]

- **Space-mean (average) speed, $u$**
  - We have: $d = s \times t$ (distance = speed $\times$ time) so $s = d / t$
  - Then
    \[
    \text{Space-mean speed} \ u = \frac{\text{Total distance travelled in } T \text{ time units}}{\text{Total time taken to traverse that distance}}
    \]

- Unless otherwise stated, when engineers talk about average speeds, they mean space-mean speeds
Vehicle travels $L$ distance units in $t_2 - t_1$ time units. Contributes $(t_2 - t_1)/L$ to space-mean speed.
It’s important to realize that space-mean and time-mean speeds will usually differ.

To see this, suppose we have a stretch of road 1.5 miles long, and we observe 4 vehicles during some particular observation period.

<table>
<thead>
<tr>
<th>Vehicle number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed speed (mph)</td>
<td>39</td>
<td>48</td>
<td>40</td>
<td>52</td>
</tr>
<tr>
<td>Travel time (mins)</td>
<td>2.31</td>
<td>1.875</td>
<td>2.25</td>
<td>1.73</td>
</tr>
<tr>
<td>Travel time (hours)</td>
<td>0.03846</td>
<td>0.03125</td>
<td>0.03751</td>
<td>0.02885</td>
</tr>
</tbody>
</table>
Then we have

\[ \text{time-mean speed} = \frac{39 + 48 + 40 + 52}{4} = \frac{179}{4} = 44.75 \text{ mph} \]

While

\[ \text{space-mean speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{6 \text{ miles}}{0.13607 \text{ hrs}} = 44.095 \text{ mph} \]
If

- \( q \) = average traffic volume (PCE’s)
- \( k \) = average traffic concentration (PCE’s / unit road length)
- \( u \) = space-mean speed

Then the Fundamental Relation says:

\[ q = ku \]
At $k = 0$ there are no vehicles /mile on the road, so the average flow ($q$) is zero.

At $k = k_{\text{max}}$ traffic stops. Hence the flow past a given point ($q$) is also zero.

Between these two extremes, flow first increases, then falls.
Suppose we observe concentration $k_1$

Then we can read off the average flow as $q_1$

Now consider the slope of line 0A: this is rise \( \div \) run = $q_1 / k_1$

By Fundamental Relation $q = ku$, this slope is the space-mean speed.
As concentration increases, speeds (slopes of the lines) fall
Speeds and Volumes

\[ q \quad u \]

\[ k \quad q_{\text{max}} \quad 0 \quad q_1 \quad q_2 \quad u_1 \quad u_2 \quad u_3 \quad u_4 \]

\[ 0 \quad k \quad k_{\text{max}} \]

\[ u \quad q \]

\[ u_1 \quad u_2 \quad u_3 \quad u_4 \quad q_1 \quad q_2 \]
Level of Service I

This is a description of a road’s traffic-related performance

- **LOS-A**: Free flow. Drivers can choose their speeds
  - For an expressway, speeds of 50+ mph
- **LOS-B**: Stable flow. Speeds restricted slightly by traffic
  - Expressways: 45–50 mph
- **LOS-C**: Stable flow: speeds *controlled* by traffic.
  - This is the design standard for US urban expressways
  - Expressways: 40–45 mph
- **LOS-D**: Approaching unstable flow
  - Expressway speeds: 35–40 mph
- **LOS-E**: Unstable flow. Speeds vary considerable. Volume-to-capacity ratio ≈ 1
  - Expressways: 30–35 mph
- **LOS-F**: Forced flow. Intermittent traffic stoppages
  - Expressways: below 30 mph
Note that LOS-F corresponds to the backward-bending part of the speed-versus-volume diagram.
Empirical Measurement

- We turn now to empirical measures of the relation between traffic volume, road capacity, and (space-mean) speeds.
- This is usually done in terms of the volume-to-capacity ratio, \( \frac{q}{c} \).
Approach I — The BPR Function

- Developed by the US Bureau of Public Roads
- Intended to be able to describe a wide variety of roads and circumstances
- Functional form:

\[
\text{Travel time per mile } t = \gamma + \beta \left( \frac{q}{c} \right)^\kappa
\]

Where:
- Travel time \( t \) is in minutes per mile
- \( q \) = traffic flow (PCE’s)
- \( c \) = road capacity (PCE’s per minute/mile)
- \( \gamma, \beta, \kappa \) are parameters
Travel time per mile \(= \gamma + \beta \left( \frac{q}{c} \right)^\kappa \)

- Suppose *no traffic* \((q = 0)\)
  - Then Travel time per mile \(t = \gamma\)
  - So \(\gamma\) can be interpreted as the free-flow travel time per mile

- Conversely, suppose the volume-to-capacity ratio is 1: \((q/c = 1)\)
  - Then Travel time per mile \(t = \gamma + \beta\)
  - So \(\gamma + \beta\) is the travel time per mile corresponding roughly to LOS-E

- The parameter \(\kappa\) is empirically estimated, and varies by type of road.
  - Observation suggests \(\kappa = 4\) for an urban expressway; \(\kappa = 2.5\) for an urban arterial
Consider a road on which we expect the free-flow speed to be 65 mph, and when the road is at capacity \((q/c = 1)\) we expect traffic to move at 25 mph.

We have
\[
\gamma = 65 \text{ mi/hr} = \frac{65 \text{ mi}}{60 \text{ min}} = \frac{60 \text{ min}}{65 \text{ mi}} = 0.92 \text{ min/mi}
\]

And
\[
\gamma + \beta = 25 \text{ mi/hr} = \frac{60 \text{ min}}{25 \text{ mi}} = 2.4 \text{ min/mi}
\]

So
\[
\gamma + \beta = 2.4
\]
\[
\beta = 2.4 - \gamma = 1.48
\]

Finally:

Travel time per mile = \(0.92 + 1.48 \left(\frac{q}{c}\right)^\kappa\)

To convert travel time \(t\) in mins/mile to speed in mph, use \(u = 60/t\)
Consider a 4-lane expressway, with per-lane capacity = 2000 PCE’s/hr.

So \( c = 2000 \times 4 = 8000 \)

Take free-flow speed = 65 mph, speed at \( q/c = 1 \) to be 25 mph

Then \( \gamma = 0.92 ; \beta = 1.48 \) (as we’ve just computed)

For an expressway, take \( \kappa = 4 \)

<table>
<thead>
<tr>
<th>( q )/lane</th>
<th>( q )</th>
<th>( q/c )</th>
<th>( t )</th>
<th>( u, \text{mph} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>2000</td>
<td>0.25</td>
<td>0.926</td>
<td>64.8</td>
</tr>
<tr>
<td>1000</td>
<td>4000</td>
<td>0.50</td>
<td>1.013</td>
<td>59.3</td>
</tr>
<tr>
<td>1500</td>
<td>6000</td>
<td>0.75</td>
<td>1.388</td>
<td>43.2</td>
</tr>
<tr>
<td>1600</td>
<td>6400</td>
<td>0.80</td>
<td>1.526</td>
<td>39.3</td>
</tr>
<tr>
<td>1700</td>
<td>6800</td>
<td>0.85</td>
<td>1.693</td>
<td>35.4</td>
</tr>
<tr>
<td>1800</td>
<td>7200</td>
<td>0.90</td>
<td>1.891</td>
<td>31.7</td>
</tr>
<tr>
<td>1900</td>
<td>7600</td>
<td>0.95</td>
<td>2.125</td>
<td>28.2</td>
</tr>
</tbody>
</table>
Approach II — Statistical Estimation

- For a given road, we know its (design) capacity $c$
- We observe traffic volume ($q_t$) and the associated speeds ($u_t$) at times $t$
- We then fit a curve to the data $((q_1, u_1), (q_2, u_2), (q_3, u_3), \ldots)$
- Note that according to our previous picture of speed versus volume, we want to fit a curve to the data, not a straight line
Roads: Several expressways in Oakland, California
Data: collected by the California Division of Highways in 1970
Estimation: Keeler & Small (1977)

Assumed functional form:

\[
\frac{q_t}{c} = \alpha_0 + \alpha_1 u_t + \alpha_2 u_t^2
\]

\(c\) is assumed known: Keeler & Small took \(c = 2000\) (PCE’s/lane/hour)

The \(\alpha\)'s are to be estimated. Note that this is linear-in-parameters
Starting from

\[
\frac{q_t}{c} = \alpha_0 + \alpha_1 u_t + \alpha_2 u_t^2
\]

We can invert to find a relation for speeds. In general we have

\[
\frac{u_t}{\alpha_2} = \left(\frac{-\alpha_1}{2\alpha_2}\right) \pm \sqrt{\frac{\alpha_1^2}{4\alpha_2^2} - \frac{1}{\alpha_2} \left(\alpha_0 - \frac{q_t}{c}\right)}
\]

If we ignore the possibility of LOS-F then we take the positive branch

Some estimation results:

<table>
<thead>
<tr>
<th>Road</th>
<th>(\hat{\alpha}_0)</th>
<th>(\hat{\alpha}_1)</th>
<th>(\hat{\alpha}_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastshore Freeway</td>
<td>-3.153</td>
<td>0.1767</td>
<td>-0.001923</td>
</tr>
<tr>
<td>Bayshore Freeway</td>
<td>-1.430</td>
<td>0.1047</td>
<td>-0.001187</td>
</tr>
<tr>
<td>Nimitz Freeway</td>
<td>-0.588</td>
<td>0.1455</td>
<td>-0.002404</td>
</tr>
</tbody>
</table>
Take the case of the Eastshore Freeway.

Then:

\[
\frac{-\alpha_1}{2\alpha_2} = \frac{-0.1767}{2 \times (-0.001923)} = 45.94 \\
\alpha_2 = \frac{(-0.1767)^2}{4(-0.001923)^2} = 2110.84 \\
\frac{1}{\alpha_2} = \frac{1}{-0.001923} = -520.02
\]

And:

\[
u_t = 45.94 + \sqrt{2110.84 + 520.02 \left(-3.153 - \frac{q_t}{c}\right)}
\]

And after a certain amount of cleaning up:

\[
u_t = 45.94 + \sqrt{471.22 - 0.26 \frac{q_t}{w}}
\]

where we have used \(c = 2000w\), where \(w\) is the number of lanes in a given direction.
For the Eastshore Freeway we have

<table>
<thead>
<tr>
<th>$q_t/w$</th>
<th>$u_t$, mph</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>64.4</td>
</tr>
<tr>
<td>1000</td>
<td>60.5</td>
</tr>
<tr>
<td>1500</td>
<td>54.9</td>
</tr>
<tr>
<td>1600</td>
<td>53.4</td>
</tr>
<tr>
<td>1700</td>
<td>51.3</td>
</tr>
<tr>
<td>1800</td>
<td>47.7</td>
</tr>
</tbody>
</table>
Moving Forward

- Now that we understand the mechanics, we are in a position to ask: what has gone wrong?
- Why are (peak) speeds so low, and why is there so much peak traffic?
- Why will the suggested fixes not work (at least in the long run)?
Sources

- **Highway Research Board.**
  “Highway capacity manual”.

- **Theodore E. Keeler and Kenneth A. Small.**
  “Optimal peak-load pricing, investment and service levels on urban expressways”.