Introduction

Understanding the Congestion Problem:

Externalities

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Congestion results from individual decisions to use the provided road infrastructure.

We already know how traffic volume and road capacity determine congestion levels (as measured by speeds).

So we turn now to the other part of the picture, to individuals’ decisions.

Setting

We consider individuals deciding whether to use a freeway in getting from origin to destination.

Each individual can decide to use the freeway, or can use a next-best alternative: this can be another road, or another mode etc.

Individuals make their decisions by comparing the cost of using the freeway with the cost of the next-best alternative. These costs can be both out-of-pocket costs and time costs.

Because individuals differ in their preferences and opportunities, each person has a different cost associated with the next-best option.

Write the cost of the next-best (non-freeway) option for individual \(i\) as \(c_{a_i}\).

Freeway Demand by Individual 1

If the freeway cost is more than \(c_{a_1}\) then no freeway trip will be made by this individual.

If the freeway cost is less than \(c_{a_1}\) then exactly one freeway trip will be made.

Individual 1’s demand for freeway travel is a step-function.
If freeway cost is more than \( c_{a2} \) no freeway trip will be made
If freeway cost is less than \( c_{a2} \) one trip will be made
Demand is again a step function

If freeway cost is more than \( c_{a1} \) neither individual will use the freeway: demand = 0
If freeway cost is between \( c_{a1} \) and \( c_{a2} \) then individual 1 will use the freeway: demand = 1
If freeway cost is less than \( c_{a2} \) then individual 2 will also use the freeway: demand = 2
Aggregate demand is a step function

With many individuals we have the same story: as the freeway cost continues to fall, more and more individuals decide to use the freeway: each time they do, demand expands by one unit.
We associate each individual with his/her alternative cost, and order the individuals.
Aggregate demand is a step function

With many individuals, the steps become smaller and smaller
In the limit, we can approximate the aggregate demand by a straight line
Note that each point on the \( Q \) axis represents the contribution by a single individual to aggregate demand.
A Property of Demand

- For any individual, the demand curve shows us that individual’s next-best cost of travel (i.e., the cost of this individual’s using the next-best alternative).
- In the picture, we show this for individual 7: the next-best cost is $c_{a7}$.

Equilibrium I

Equilibrium is a setting in which some number (say, $Q^*$) of individuals use the freeway, and no-one has an incentive to switch to the alternative travel option.

Equilibrium II

- $AC(Q)$ is the average user cost for freeway users, if $Q$ people decide to use the freeway.
- Because of congestion, $AC$ is an increasing function of traffic.

Equilibrium III

- Claim: the equilibrium traffic on the freeway is $Q^*$, where the demand function $D$ intersects the average-user-cost function $AC$.
- At traffic $Q^*$, each freeway user experiences a cost of $AC(Q^*)$.
- To verify that this is the equilibrium, we need to check that no freeway user has an incentive to switch to the alternative.
Consider individual 1 ($Q_1$), who is a freeway user.

- Her cost of using the alternative is $c_{a1}$, from the demand function
- This is greater than $AC(Q^*)$ so this person has no incentive to switch.
- The same thing applies to any freeway user (all points to the left of $Q^*$)

Now consider individual 2 ($Q_2$), who uses her alternative means of travel.

- The cost of using her alternative is $c_{a2}$, from the demand function
- This is less than $AC(Q^*)$ so this person has no incentive to switch.
- The same thing applies to any non-freeway-user (all points to the right of $Q^*$).
- So no-one has an incentive to switch, and $Q^*$ is the equilibrium.

We define the (transportation) optimum as the freeway usage that minimizes the total travel costs of the travelling population.

- Note that the optimum doesn’t involve individual decision-making: it as if the planner assigns different individuals either to the freeway or to that individual’s next-best alternative.

Suppose we have $Q$ people using the freeway.

- We can read off the average user cost of each of these individuals as $AC(Q)$. The total user costs of the freeway users is therefore $TC(Q) = Q \cdot AC(Q)$
- Now suppose we re-assign one of the freeway users to his or her alternative.
- Freeway usage goes down by one unit, to $Q - 1$
- The change (decrease) in the total costs of the freeway users is $TC(Q) - TC(Q - 1)$
- This is the marginal user cost $MC(Q)$ of the freeway users.
Conversely, suppose that, starting from $Q$ users, we re-assign a non-freeway user to the freeway. The change (increase) in the total costs of the freeway users is $\text{TC}(Q + 1) - \text{TC}(Q)$. This is also the marginal user cost $MC(Q)$ of the freeway users. So the marginal cost $MC(Q)$ represents the change in the user costs for the freeway users resulting from a 1-unit change in traffic, in either direction. If traffic increases, $MC(Q)$ is an increase in total freeway-user costs, while if traffic falls, $MC(Q)$ represents a decrease in total freeway-user costs. It can be shows that because $AC(Q)$ is rising, then $MC(Q)$ must be above $AC(Q)$.

Claim: the optimum occurs at traffic level $Q^{**}$, where the marginal cost curve intersects the demand curve. To verify this, we need to check whether re-assigning one individual more or less to the freeway can decrease total costs. If no such assignment can decrease costs, then $Q^{**}$ is indeed the optimum.

Consider individual 1, currently assigned to the freeway. If we assigned her to her alternative, then on the freeway side we’d reduce costs by $MC(Q^{**})$. But on the other hand, she would incur cost $c_{a1}$. Since $c_{a1}$ is greater than $MC(Q^{**})$, assigning individual 1 to the non-freeway wouldn’t decrease total costs.

Consider individual 2, currently assigned to the non-freeway mode. If we assigned her to the freeway, then on the freeway side we’d increase costs by $MC(Q^{**})$.

But on the other hand, she would reduce her costs by $c_{a2}$. But since $c_{a2}$ is less than $MC(Q^{**})$, re-assigning individual 1 to the freeway would have the net effect of increasing total costs.
So, starting from traffic $Q^{**}$ on the freeway, we cannot re-assign travellers to other modes in such a way as to reduce total costs.

Therefore $Q^{**}$ is the optimum assignment of travellers to the freeway.

Comparing $Q^{**}$ (the optimum) and $Q^{*}$ (the equilibrium) we see that too many people use the freeway.

This is precisely the problem of traffic congestion.

Essentially, the problem is that in making their own individual decision as to which travel option to use, people base their decisions on their private costs (AC) rather than on total community costs (MC).

They ignore the impact that their decision has on other users.

This is called an *externality*: a setting in which:
- my decision has an impact on others; and
- I have no incentive to take that impact-on-others into account

In this case, because my decision causes other people’s costs to increase, we have a *negative externality*.

You can think of cases where my action generates a benefit to someone else: this is a positive externality.

Bottom line: in order to minimize total (community) travel costs, individuals should base their decisions on marginal, and not average cost.

Consider an individual thinking of making a trip in the peak, but who before our proposed solutions *just* decides not to (In other words, this person’s net benefit from the trip is *just* negative)

The proposed solutions *all have* the effect of reducing traffic flow

This reduces the cost for our individual (she sees that speeds have increased)

So she will now change her mind and make the trip.

Obviously, this applies to every individual in the urban area

So if we implement any of these ideas we would expect to see an initial speed-up of traffic, followed by a return to the *status quo ante* as previously “discouraged” travellers change their minds and/or their routes.

One exception: if there are no on-the-fence travellers, then the proposed solutions can work.
So What Will Work?

- If the conventional solutions won’t solve the problem, what will?
- We already have a diagnosis of the problem: it’s an *incentives* problem, namely that people are basing their decisions on their average costs \( AC(Q) \) rather than the marginal cost \( MC(Q) \).
- Can we exploit this understanding of the problem in order to formulate a solution that will actually work?

Average vs Marginal Costs I

- We are interested in comparing average and marginal costs of freeway travel. Remember that:
  \[
  TC(Q) = Q \cdot AC(Q)
  \]
- So
  \[
  MC(Q) = \frac{d}{dQ} TC(Q) = \frac{d}{dQ} Q \cdot AC(Q) = AC(Q) + Q \frac{d}{dQ} AC(Q)
  \]

Average vs Marginal Costs II

- This gives us relation between the things we are interested in, the marginal cost and the average cost.
- The difference between what we want people to use in their decision-making \( MC \) and what they do use \( AC \) is:
  \[
  MC(Q) - AC(Q) = Q \frac{d}{dQ} AC(Q)
  \]

Average vs Marginal Costs III

- We are looking at the difference between marginal and average costs:
  \[
  \tau = Q \frac{d}{dQ} AC(Q)
  \]
- First, we have \( (dAC/dQ) \) : this term represents the additional cost that one more freeway user imposes on each existing user
- This term is multiplied by the number \( Q \) of existing users.
- So \( \tau \) is is the total cost that one more freeway user imposes on all other (existing) freeway users.
If people faced the MC of their decisions as the relevant cost, then they would make the right decisions.

In order to bring this about we need to add the term

$$\tau = Q \frac{d}{dQ} AC(Q)$$

to their user-perceived average cost.

This term is the *congestion toll*: the additional cost people need to face in order to get them to do the right thing (where “right thing” is to adopt the behavior that minimizes total costs of travel, which we used to define the optimum).

So our answer is that we need to charge people the appropriate congestion toll, in order to get them to make the (socially) correct decisions.

But before we go into more detail, let’s expand the scope of our analysis.

We’ve implicitly assumed that the size of the freeway was given. But we can quite reasonably ask — even though we know it probably won’t solve the congestion problem — how much roadway capacity should be provided?

It would be interesting to know if we have “too much” roadway, or if we really should be building more roads.

Once we’ve answered that, then we can look in more detail at the structure of congestion tolls.

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**Reference**

Much of this material is based on: