Logit and WTP

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Introduction

A transportation planner proposes a change in the transport environment (changes transit frequency or fare; changes tolls or gasoline taxes; or perhaps several of these at once).

Question: how does this affect the transportation users?

Because we will often want to balance that impact against the implementation costs, we seek a money measure of the impact.

We ask: what is an individual willing to pay (wtp) for the change, when that change is perceived as a benefit; and what is the individual willing to pay to avoid the change, when the change is perceived as a dis-benefit.

In the remainder of this note we develop the **Compensating Variation** as a useful money measure of wtp.

We first develop it in a non-transportation context (continuous choice, for a price change only) and we then apply it to the discrete-choice logit setting (where the change may be more than just price).

Compensating Variation: Continuous Choice

Setup:

- Conventional setting (ie not discrete choice): wtp for a price change
- Individual in a 2-good world:
  - good \( x \) with price \( p_x \)
  - good \( y \) (= everything else) with price 1
  - income \( M^0 \)
- Experiment: we are interested in the individual's wtp for a price reduction for good 1, where the price changes from \( p_x^0 \) (initial price) to \( p_x^1 \) (final price)
Compensating Variation: Initial Position

- Prices are \((p^0_x, 1)\)
- Initial equilibrium at \(A\)
- Achieves maximum utility \(u^0\)

Compensating Variation: Final Position

- Price of \(x\) falls to \(p^1_x\) so individual faces prices \((p^1_x, 1)\)
- New equilibrium at \(B\)
- Achieves maximum utility \(u^1\) (better)

Compensating Variation I

- With prices at \(p^1_x\) we take away income and restore individual to \(u^0\). This is bundle \(C\)
- New income: \(M^1\)
- Compensating variation \(CV = M^0 - M^1 > 0\)

Compensating Variation I

Compare bundles \(A\) and \(C\):

- \(C\) involves
  - \(CV = M^0 - M^1\) less income than \(A\)
- \(C\) involves lower \(p_x\) than \(A\)
- But the individual regards \(A\) and \(C\) as equally good
- So the decrease in income exactly balances the benefit of the lower price
- Therefore the value of the benefit of the lower price is exactly \(CV\)
Compensating Variation II

- CV represents the individual’s WTP for the price change, given that it has been implemented. It takes the post-change position (B) as the basis for the compensation/restoration.
- This amounts to an ex-post project evaluation.
- The other possibility is to take the pre-change position as the basis. This gives rise to the Equivalent Variation (EV). It is appropriate for an ex-ante project evaluation.
- Note that there is a general implementation difficulty here, since the CV requires us to know the two utility levels — which we usually don’t.
- Instead, we generally compute the change in consumer’s surplus, and regard it as an approximation to the CV or EV.

Compensating Variation and Logit I

- We generalize the CV idea by allowing the modal characteristics $x_{ij}$ to change from initial values $x_{ij}^0$ to final values $x_{ij}^1$.
- For the logit model we have an explicit formula:

$$CV = -\frac{1}{\beta_C} \left[ \ln \sum_{j=1}^{J} e^{x_{ij} \beta} \right]_{x_{ij}=x_{ij}^0} - x_{ij}^1$$

where:

- the notation $[f(x)]_{x=a}^{b}$ means: evaluate the quantity in the brackets at a (to get $f(a)$), then at b (giving $f(b)$), and subtract: $f(a) - f(b)$
- $\beta_C$ is the coefficient of cost or price in the systematic portion of utility.

CV Example: Logit Model

- Individual faces a choice of 4-modes: auto-alone; bus + walk access; bus + auto access; carpool.
- Post-tax income: $50,000 per year = 40.8479 \$/min
- Demand model: model 12 from McFadden + Talvitie (1978) estimated for SFBA work trips.
- Results: naive model (few independent variables)

<table>
<thead>
<tr>
<th>Indep var.</th>
<th>Estd. Coeff</th>
<th>t-stat.</th>
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</thead>
<tbody>
<tr>
<td>Cost/post-tax-wage ($/min)</td>
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<td>7.63</td>
</tr>
<tr>
<td>In-vehicle time (mins)</td>
<td>-0.0201</td>
<td>2.78</td>
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<tr>
<td>Excess time (mins)</td>
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<td>Auto dummy</td>
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<tr>
<td>Bus+Auto dummy</td>
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<tr>
<td>Carpool dummy</td>
<td>-2.15</td>
<td>8.56</td>
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CV Example: Initial Conditions

<table>
<thead>
<tr>
<th></th>
<th>Auto</th>
<th>Bus+Walk</th>
<th>Bus+Auto</th>
<th>Carpool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost/post-tax wage</td>
<td>2.448</td>
<td>3.06</td>
<td>3.06</td>
<td>1.224</td>
</tr>
<tr>
<td>In vehicle time</td>
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<td>30</td>
<td>30</td>
<td>20</td>
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<tr>
<td>Excess time</td>
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<td>5</td>
<td>10</td>
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<tr>
<td>Auto dummy</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Bus+Auto dummy</td>
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<td>1</td>
<td>0</td>
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<tr>
<td>Carpool dummy</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Raw Costs/Fares</td>
<td>1.00</td>
<td>1.25</td>
<td>1.25</td>
<td>0.50</td>
</tr>
<tr>
<td>Choice Probabilities</td>
<td>0.3888</td>
<td>0.4445</td>
<td>0.0979</td>
<td>0.0683</td>
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</tbody>
</table>

We assume that car-pooling involves 2 people and they split the auto costs.

CV Example: Final Conditions

Suppose the bus-transit provider reduces the bus fare to $1.05 and decreases in-vehicle travel time to 25 mins

<table>
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<th>Carpool</th>
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<td>Cost/post-tax wage</td>
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<tr>
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<tr>
<td>Bus+Auto dummy</td>
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<tr>
<td>Carpool dummy</td>
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<td>1</td>
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<tr>
<td>Raw costs/Fares</td>
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<td>1.05</td>
<td>0.50</td>
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<tr>
<td>Choice Probabilities</td>
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<td>0.4693</td>
<td>0.1032</td>
<td>0.0639</td>
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</tbody>
</table>

CV Example: Calculations I

\[
\begin{array}{c|cccc}
\times \beta : \text{initial} & -1.39486 & -1.26007 & -2.77457 & -3.13343 \\
\times \beta : \text{final} & -1.39486 & -1.13940 & -2.65390 & -3.13343 \\
\end{array}
\]

Then:

- Logsums \((\ln \sum e^{x_i/\beta})\):
  - Initial: \(-0.450285\)
  - Final: \(-0.382984\)

\[\beta_C = \frac{-0.0412/40.8479}{\beta_{wage}} = -1.0086 \times 10^{-3} \] (since we have \[\beta_{wage} \times \text{cost}\], so the coefficient of cost is \[\frac{\beta}{\text{wage}}\]).

CV Example: Calculations II

- Difference in logsums: \(-0.382984 - (-0.450285) = 0.067301\)
- \[CV = \left(-\frac{1}{\beta_C}\right) \times 0.067301 = \left(-\frac{1}{-0.0010086}\right) \times 0.067301 = 66.727\]
- Final result: Compensating Variation = 66.7268¢ (result is in cents since costs are in cents).
- This individual is willing to pay about 67¢ for the reduction in fare and the improvement in travel time on the bus mode.