Logit and WTP

Philip A. Viton

February 23, 2012
A transportation planner proposes a change in the transport environment (changes transit frequency or fare; changes tolls or gasoline taxes; or perhaps several of these at once).

Question: how does this affect the transportation users?
Because we will often want to balance that impact against the implementation costs, we seek a money measure of the impact.

We ask: what is an individual *willing to pay* (wtp) for the change, when that change is perceived as a benefit; and what is the individual willing to pay to *avoid* the change, when the change is perceived as a dis-benefit.

In the remainder of this note we develop the *Compensating Variation* as a useful money measure of wtp.

We first develop it in a non-transportation context (continuous choice, for a price change only) and we then apply it to the discrete-choice logit setting (where the change may be more than just price).
Compensating Variation: Continuous Choice

Setup:

- Conventional setting (ie not discrete choice): wtp for a price change
- Individual in a 2-good world:
  - good $x$ with price $p_x$
  - good $y$ (= everything else) with price 1
  - income $M^0$
- Experiment: we are interested in the individual’s wtp for a price reduction for good 1, where the price changes from $p_x^0$ (initial price) to $p_x^1$ (final price)
Prices are \((p_x^0, 1)\)

- Initial equilibrium at \(A\)
- Achieves maximum utility \(u^0\)
Compensating Variation: Final Position

- Price of $x$ falls to $p^1_x$ so individual faces prices $(p^1_x, 1)$
- New equilibrium at $B$
- Achieves maximum utility $u^1$ (better)
With prices at $p_x^1$ we take away income and restore individual to $u^0$. This is bundle C

- New income: $M^1$
- Compensating variation $CV = M^0 - M^1 > 0$
Compare bundles $A$ and $C$:

- $C$ involves
  \[ CV = M^0 - M^1 \]
  less income than $A$

- $C$ involves lower $p_x$ than $A$

- But the individual regards $A$ and $C$ as equally good

- So the decrease in income exactly balances the benefit of the lower price

- Therefore the value of the benefit of the lower price is exactly $CV$
CV represents the individual’s wtp for the price change, *given that it has been implemented*. It takes the post-change position \(B\) as the basis for the compensation/restoration.

This amounts to an *ex-post* project evaluation.

The other possibility is to take the pre-change position as the basis. This gives rise to the Equivalent Variation (EV). It is appropriate for an *ex-ante* project evaluation.

Note that there is a general implementation difficulty here, since the CV requires us to know the two utility levels — which we usually don’t.

Instead, we generally compute the change in consumer’s surplus, and regard it as an approximation to the CV or EV.
We generalize the CV idea by allowing the modal characteristics $x_{ij}$ to change from initial values $x_{ij}^0$ to final values $x_{ij}^1$.

For the logit model we have an explicit formula:

$$CV = -\frac{1}{\beta_C} \ln \left[ \sum_{j=1}^{J} e^{x_{ij} \beta} \right]_{x_{ij}=x_{ij}^1}^{x_{ij}=x_{ij}^0}$$

where:

- the notation $[f(x)]_{x=a}^{x=b}$ means: evaluate the quantity in the brackets at $a$ (to get $f(a)$), then at $b$ (giving $f(b)$), and subtract: $f(a) - f(b)$
- $\beta_C$ is the coefficient of cost or price in the systematic portion of utility.
We are able to derive a formula for the logit model because, as we’ve seen, we are estimating the systematic part of utility. This is unusual in applied work. It turns out that for the logit model, the ex-ante and ex-post comparisons (ie, the CV and EV) are the same. For more on this, see Small + Rosen (1981)
Individual faces a choice of 4-modes: auto-alone; bus + walk access; bus + auto access; carpool

Post-tax income: $50,000 per year = 40.8479 \( \$ \)/min

Demand model: model 12 from McFadden + Talvitie (1978) estimated for SFBA work trips

Results: naive model (few independent variables)

<table>
<thead>
<tr>
<th>Indep var.</th>
<th>Estd. Coef</th>
<th>t-stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost/post-tax-wage (( $ ) ( \div ) ( $ )/min)</td>
<td>-0.0412</td>
<td>7.63</td>
</tr>
<tr>
<td>In-vehicle time (mins)</td>
<td>-0.0201</td>
<td>2.78</td>
</tr>
<tr>
<td>Excess time (mins)</td>
<td>-0.0531</td>
<td>7.54</td>
</tr>
<tr>
<td>Auto dummy</td>
<td>-0.892</td>
<td>3.38</td>
</tr>
<tr>
<td>Bus+Auto dummy</td>
<td>-1.78</td>
<td>7.52</td>
</tr>
<tr>
<td>Carpool dummy</td>
<td>-2.15</td>
<td>8.56</td>
</tr>
</tbody>
</table>
We assume that car-pooling involves 2 people and they split the auto costs.
Suppose the bus-transit provider reduces the bus fare to $1.05 and decreases in-vehicle travel time to 25 mins.

<table>
<thead>
<tr>
<th></th>
<th>Auto</th>
<th>Bus+Walk</th>
<th>Bus+Auto</th>
<th>Carpool</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost/post-tax wage</strong></td>
<td>2.448</td>
<td>2.5704</td>
<td>2.5704</td>
<td>1.224</td>
</tr>
<tr>
<td><strong>In vehicle time</strong></td>
<td>20</td>
<td>25</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td><strong>Excess time</strong></td>
<td>0</td>
<td>10</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td><strong>Auto dummy</strong></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Bus+Auto dummy</strong></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td><strong>Carpool dummy</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Raw costs/Fares</strong></td>
<td>1.00</td>
<td>1.05</td>
<td>1.05</td>
<td>0.50</td>
</tr>
<tr>
<td><strong>Choice Probabilities</strong></td>
<td>0.3635</td>
<td>0.4693</td>
<td>0.1032</td>
<td>0.0639</td>
</tr>
</tbody>
</table>

Philip A. Viton
CV Example: Calculations I

<table>
<thead>
<tr>
<th></th>
<th>Auto</th>
<th>Bus+Walk</th>
<th>Bus+Auto</th>
<th>Carpool</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x\beta$: initial</td>
<td>-1.39486</td>
<td>-1.26007</td>
<td>-2.77457</td>
<td>-3.13343</td>
</tr>
<tr>
<td>$x\beta$: final</td>
<td>-1.39486</td>
<td>-1.13940</td>
<td>-2.65390</td>
<td>-3.13343</td>
</tr>
</tbody>
</table>

Then:

- **Logsums** ($\ln \sum_j e^{xij\beta}$):
  - Initial: $-0.450285$
  - Final: $-0.382984$

- $\beta_C = -0.0412/40.8479 = -1.0086 \times 10^{-3}$ (since we have $\beta_{\text{cost}} = \frac{\beta}{\text{wage}} \times \text{cost}$, so the coefficient of cost is $\frac{\beta}{\text{wage}}$).
Difference in logsums: $-0.382984 - (-0.450285) = 0.067301$

$CV = \left(-\frac{1}{\beta_C}\right) \times 0.067301 = \left(-\frac{1}{0.0010086}\right) \times 0.067301 = 66.727$

Final result: Compensating Variation $= 66.7286 \, \mathcal{C}$ (result is in cents since costs are in cents).

This individual is willing to pay about $67\mathcal{C}$ for the reduction in fare and the improvement in travel time on the bus mode.