The Monocentric City Model

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Some Facts Needing Explanation

- As we get further away from an activity center, land rents decline.
- As we get further away from an activity center, lot sizes increase.
- As we get further away from an activity center, population density falls.
- In the US at least, we observe a good deal of spatial segregation by class: the rich tend to live in the suburbs while the poor tend to live in the central cities.

Can we use our understanding of individual behavior to explain these facts?
All locations are on a flat featureless von Thunen plain. (This means that geography — rivers, mountains etc — is irrelevant).

There is a small city in the midst of the plain. The city is made up of:

- A single central business district (CBD) which we will take to be a point. This defines the *monocentric* city.
- Suburbs surrounding the CBD.

Around the city is a “hinterland” devoted to agriculture.

We measure all distances $s$ as distances from the CBD.
Model: Economy

Urban sectors:

- Land sector: we denote the lot size in the city by \( q \).
- City transport sector: a single mode of transport is available in any direction.
- Goods sector: besides land, there is a single composite good \( z \).

Prices

- City land price (called the land *rent*) at distance \( s \) from the CBD is \( R(s) \).
- Agricultural land rent is \( R_A \) everywhere in the hinterland.
- Transport: available at a cost of \( k \) per round-trip mile.
- The composite good is available everywhere (it is *ubiquitous*) at a price of 1.
Individuals

- All individuals are utility-maximizing price-takers.
- All city dwellers, no matter where they live, must commute to the CBD to earn income.
- Individuals can migrate in and out of the city at zero cost: this defines the *open* monocentric city.
The 1-Class City

- All individuals are identical with the same utility functions and income $M$. We also assume that they are identical in all other aspects, e.g., family size. They really are clones.
- If an individual lives at distance $s$ from the CBD she incurs commute costs of $ks$.
- Therefore her *disposable income* available to purchase land and $z$ (at distance $s$ from the CBD) is $Y(s) = M - ks$. 
In equilibrium each individual must be on the same indifference curve (no matter where they live), which we refer to as the spatial-equilibrium indifference curve $I^*$. 

Reason: if not, then the person on the lower indifference curve could move and imitate the consumption bundle of a person on a higher indifference curve. This is possible since everyone has the same tastes and income.

So in the 1-class city, having people on different indifference curves cannot be an equilibrium.
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Diagram of the 1-Class Monocentric City
Let $I^*$ be the common spatial-equilibrium indifference curve.

Suppose an individual lives at $s_1$.

Then disposable income there is $Y(s_1) = M - ks_1$. This is also the maximum quantity of the composite good $z$ that this person can afford (since $p_z \equiv 1$ everywhere).

The budget constraint of the person living at $s_1$ therefore:

- Goes through $Y(s_1)$.
- Is tangent to $I^*$.

This determines the budget constraint as line $A$.

The slope of the budget constraint $A$ is

$$-[\text{price of land at } s_1] \div [\text{price of } z] = -R(s_1).$$
Now consider an individual living closer to the CBD (at $s_2$).

She has disposable income $Y(s_2)$.

By the same reasoning, her budget constraint is line $B$, also tangent to $l^*$.

Slope $= -R(s_2)$. 
Clearly, line $A$ (the budget constraint at distance $s_1$) is less steep than line $B$ (the budget constraint at distance $s_2$, which is nearer to the CBD).

We therefore conclude:

Rents decrease with distance from the CBD.
Lot Sizes, Density

- From the diagram, the lot size at $s_1 \ (= q(s_1) \ )$ is larger than the lot size at $s_2 \ (= q(s_2) \ )$.
- We conclude that lot sizes increase with distance from the CBD. Of course this makes sense since land prices decline with distance.
- We also conclude that density declines with distance. (since by our assumption all individuals/families are the same size, and lot sizes are larger away from the CBD.)
Two classes of individuals:

- Rich, with incomes $M_R$.
- Poor, with income $M_P$.
- Obviously, $M_R > M_P$.

Within each class, identical individuals. But individuals’ tastes (utility functions) may differ as between the two classes.

All use the same transport system.

Land is a normal good: as incomes go up (holding everything else constant) people want to consume more land.
We now have representatives from two classes potentially interested in the same plot of land.

Who gets the land?

Obvious principle: land goes to the person who bids higher for it.

So we need to see how much each class (ie a representative person of that class) will bid for land (at a given distance).
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Bid Rent

\[ Y(s) \]

\[ z, \$ \]

\[ Y(s_1) \]

\[ A \]

\[ B \]

\[ C \]

\[ I^* \]

\[ s_1 \]
Consider a person living $s_1$ miles from the CBD.

What is the difference between budget constraints $A$, and $C$?

Answer: because it is steeper, $C$ represents a higher land price.

So to find the maximum bid for land for someone living at $s_1$ consistent with spatial equilibrium, we find the steepest line through $Y(s_1)$ that is just tangent to the spatial equilibrium indifference curve $I^*$. In the picture, this is line $B$.

But of course, this is what we have been working with all along.

The maximum bid for land (at some distance) that is consistent with spatial equilibrium is called the Bid Rent of land (at that distance).
We have 2 downward-sloping bid-rent curves.

There must be one point in space \( (s^*) \) where the two groups live side-by-side. Call this the integrated district.

The question is: which bid-rent curve is that of the poor and which belongs to the rich?
2-Class City: The Integrated District

\[ Y_R(s) \]
\[ Y_P(s) \]
\[ Y_R(s^*) \]
\[ Y_P(s^*) \]
\[ z, s \]

Diagram showing the relationship between different variables in a 2-Class City model.
At $s^*$ the poor people must attain the poor-persons' spatial equilibrium indifference curve $I_p$: this determines their budget constraint ($A$).

At $s^*$ the rich pay the same rent: their budget constraint is therefore parallel to $A$.

And since at $s^*$ the rich have more disposable income than the poor, and land is normal, they therefore demand larger lot sizes than the poor.

This determines where their budget constraint is tangent to the spatial-equilibrium indifference curve of the rich, $I_R$: the result is the budget constraint $B$. 
2-Class City — Who Lives Where I

\[ Y_R(s) \]
\[ Y_p(s) \]
\[ Y_R(s_1) \]
\[ Y_p(s_1) \]
\[ z_1,s \]

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Now consider a point like $s_1$ (closer to the CBD than the integrated district at $s^*$).

- A poor person living there must have a budget constraint tangent to $I_P$: this is line $A$.
- Suppose that the rich paid the same rent as the poor at $s_1$.
- Then their budget constraint would be line $B$ (parallel to $A$).
- But this cannot be an equilibrium, since it is not tangent to $I_R$.
- In order for the budget constraint to be tangent to $I_R$ it must pivot outwards, to $C$.
- But as compared to $B$ (and $A$), line $C$ represents a lower bid-rent (at $s_1$) since it is less steep.
- We therefore conclude that the poor outbid the rich at $s_1$. 
We conclude that in our 2-class open monocentric city:

- The poor live closer to the CBD.
- The rich live in the suburbs.
The city will end when the bid-rents of the rich ($R_R(s)$) fall below the bid-rents of the farmers ($R_A$).

In the picture, this is $s_f$. 
The equilibrium pattern of land rents will therefore be the outer envelope of $R_P(s)$ (the bid-rents of the poor); $R_R(s)$ (the bid-rents of the rich); and $R_A$ (the bid rent of the farmers).

In the picture, this is the heavy line.