1. In Region 1, demand and supply for some good are:

\[ x_1^* = 15.40 - 0.42p_1 \] (Demand)
\[ x_1^* = 1.11p_1 - 16.67 \] (Supply)

(a) What is the autarkic equilibrium price and quantity in this region?
(b) At \( p_1 = 16 \) what is Region 1’s excess demand?

**Answer Sketch:** In Region 1:

(a) Equate demand and supply and solve for price:

\[ 15.40 - 0.42p_1 = 1.11p_1 - 16.67 \]
\[ 15.40 + 16.67 = 1.11p_1 + 0.42p_1 \]
\[ 32.07 = 1.53p_1 \]

So \( p_1^* = \frac{32.07}{1.53} = 20.96 \). To find the autarkic quantity, just plug this price into either the demand or supply equations: using demand we see that \( x_1^* = 15.40 - (0.42 \times 20.96) = 6.5968 \)

(b) To find the excess demand just plug in \( p_1 = 16 \) and find

Demand: \( 15.40 - (0.42 \times 16) = 8.68 \)
Supply: \( (1.11 \times 16) - 16.67 = 1.09 \)

Hence Excess Demand = Demand – Supply = 8.68 – 1.09 = 7.59

2. In Region 2, demand and supply for the same good are

\[ x_2^* = 10.00 - 0.83p_2 \] (Demand)
\[ x_2^* = 2.00p_2 - 16.00 \] (Supply)

(a) What is the autarkic equilibrium price and quantity in this region?
(b) At \( p_2 = 11.87 \) what is Region 2’s excess supply?

**Answer Sketch:** In Region 2:

(a) Equate demand and supply and solve:

\[ 10.00 - 0.83p_2 = 2.00p_2 - 16.00 \]

Hence \( p_2^* = 9.1873 \) which we may round to 9.19. Using the supply function, the equilibrium quantity is \( (2.0 \times 9.1873) - 16.00 = 2.3746 \)
(b) At \( p_2 = 11.87 \), Demand = \( 10.00 - (0.83 \times 11.87) = 0.1479 \), while Supply = \( (2.00 \times 11.87) - 16.00 = 7.74 \). Hence the excess supply is 7.74 – 0.1479 = 7.5921.

3. Assume that inter-regional trade between the two regions in the previous questions becomes possible at a transport cost of 3.50 per unit of product shipped. Assuming that trade must balance to within 0.02 units, could \( p_1 = 16 \) and \( p_2 = 11.87 \) be an inter-regional trade equilibrium?

**Answer Sketch:** From the previous two questions, we see that at these prices trade (approximately) balances: the excess demand in Region 1 is about the same as the excess supply in Region 2. But for this to be an equilibrium, the price difference must exactly equal the transport cost, here 3.50. But \( p_1 - p_2 = 16 - 11.87 = 4.13 \neq 3.50 \), so this can’t be an equilibrium.

4. A utility-maximizing, price-taking individual in a 2-good world has a utility function

\[
u(x_1, x_2) = 2.5x_1 + 3.7x_2 - 4.0x_1x_2\]

Consider the two bundles \( A = (1, 4) \) and \( B = (2, 3) \). Which does this person prefer?

**Answer Sketch:** Since the utility function represents preferences, all we need to do is compute the utilities and see which is greater.

For Bundle \( A = (1, 4) : u(x_1, x_2) = (2.5 \times 1) + (3.7 \times 4) - (4.0 \times 1 \times 4) = 1.3 \)

For Bundle \( B = (2, 3) : u(x_1, x_2) = (2.5 \times 2) + (3.7 \times 3) - (4.0 \times 2 \times 3) = -7.9 \)

So bundle \( A \) is preferred.

5. A utility-maximizing, price-taking individual faces \( p_1 = 7.50 \) and \( p_2 = 3.00 \). She is observed to consume 100 units of \( x_1 \) and 300 units of \( x_2 \). How much \( x_2 \) would we need to give her in order to persuade her to cut her consumption of \( x_1 \) by two units?

**Answer Sketch:** If she’s in equilibrium, we know that her MRS equals minus the price ratio, so

\[
\frac{\Delta x_2}{\Delta x_1} = -\frac{7.50}{3.00} = -2.5
\]

or \( \Delta x_2 = -2.5\Delta x_1 \). In this case \( \Delta x_1 = -2 \) so \( \Delta x_2 = 5 \). We’d need to give her 5 units of \( x_2 \) to persuade her to give up 1 unit of \( x_1 \). (A number of people thought that the equation was \( x_2/x_1 = -p_1/p_2 \) : remember (a) that slopes involve the changes in quantities; and (b) a symbol like \( \Delta x_2 \) is a single symbol standing for “the change in \( x_2 \)” so the \( \Delta \)’s in the numerator and denominator don’t cancel).

6. If an individual tells you that he’s *always* willing to give up 2 units of \( x_2 \) in order to get one more unit of \( x_1 \) no matter how much \( x_1 \) or \( x_2 \) he has, what does this tell you about the shape of his indifference curves?

**Answer Sketch:** If he’s *always* willing to give up 2 units of \( x_2 \) to get 1 more unit of \( x_1 \) that implies that his MRS is constant. Since the MRS is the slope of the indifference curve, we’re saying that the indifference curves have the same slope everywhere. The only curve with this property is a straight line. In this case we have \( \Delta x_2/\Delta x_1 = -2/1 = -2 \), so the indifference curves are downward-sloping straight lines, with slope \(-2\).
7. Consider the monocentric open-city model with identical individuals which we have discussed in class. Suppose that for some reason the spatial-equilibrium utility level increases. What will this do to the land rent at some arbitrary distance $s_1$ from the CBD?

**Answer Sketch:** See the picture below. The person at $s_1$ has disposable income $I(s_1)$. Hence, if $u^*$ is the original spatial equilibrium indifference curve, the budget constraint must go through $I(s_1)$ and be tangent to $u^*$: this is the line marked A. If the indifference curve were the higher one marked $u^{**}$, then we'd generate line B, by the same reasoning.

Now the slope of the budget constraint is minus the land rent. Obviously, line B is less steep than line A, which means that increasing the spatial-equilibrium utility level lowers rents at $s_1$. (You should convince yourself that this holds at any distance, including the origin).

8. The diagram — which is not drawn accurately to scale — below shows the setup for a utility-maximizing price-taking individual in a 2-good world.

(a) If she has income 100 and faces prices $p_1 = 4$ and $p_2 = 5$ what is her optimum consumption bundle?

(b) What is her optimal bundle if $p_1$ rises to 5?

(c) What does this diagram tell us about her own-price elasticity of demand at $A$?

(d) Assuming that the budget constraints through A and C are parallel, what does this diagram tell us about her income-elasticity of demand at $A$?

(In answering the last two parts, you may assume that everything is relevantly small. In the diagram, 6-2/5 means $\frac{6}{2}$, etc).

**Answer Sketch:** For this individual:
(a) If income is 100 and \( p_1 = 4 \) and \( p_2 = 5 \) then the intercepts of the budget constraint will be 100/4 = 25 on the \( x_1 \) axis and 100/5 = 20 on the \( x_2 \) axis. Then bundle A is the optimal bundle, and the quantities of \( x_1 \) and \( x_2 \) are (18, \( 5\frac{2}{3} \)).

(b) If \( p_1 = 5 \), then the \( x_1 \)-intercept becomes 100/5 = 20 and nothing else changes. This identifies point B = (16, 4) as the optimal bundle.

(c) To address the question of the own-price elasticity around A we need to find an equilibrium on a budget line for which the only change is a change in the own-price, ie a change in \( p_1 \). As we’ve just see, bundle B qualifies. Thus, when the price of \( x_1 \) rises from 4 to 5, consumption falls from 18 to 16. Hence

\[
\eta_{p_1} = \frac{\Delta x_1}{\Delta p_1} \cdot \frac{p_1}{x_1} = \frac{-4}{-1} \frac{4}{18} \frac{1}{1} = -\frac{4}{9} = -0.44444
\]

The own-price elasticity is \(-0.444\).

(d) If the budget line through C is parallel to the one through A then all that’s changed is income. Hence bundle C can be compared to bundle A to study the income elasticity of demand. To find out what the income on the budget line through C is, just compute the cost of bundle C: \((22 \times 4) + (6\frac{2}{3} \times 5) = \)
120. Then we see that when income increases by 20, consumption of \( x_1 \) increases by 4. Hence

\[
\eta_M = \frac{\Delta x_1}{\Delta M} \frac{M}{x_1} = \frac{4}{20} \frac{100}{18} = \frac{10}{9} = 1.1111
\]

So the income elasticity of demand is 1.111.